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A STUDY ON COMMUNICATION ANTENNA ISOLATION

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16. Abstract

Problems on FAA communication antennas related to isolation are analyzed. The concept of using progressively phased circular array to achieve high isolation is studied.

The method of study is divided into theoretical and experimental. Theoretical study involved using moment method. Symmetrical dipoles were used as elements. Factors analyzed include array radius, antenna size, position accuracy, impedance, radiation pattern, and isolation.

The experimental study involved building scale antennas from existing FAA coaxial dipoles and used them as array elements, and simulated matrix feed conditions to study isolation and radiation pattern. An isolation of 55 db was found possible.

The concept is shown to provide high isolation in a very limited space. It provides flexibility for beam forming, and azimuth scanning.

17. Key Words
Array, Circular Array, Isolation, Moment Method, Coaxial Dipole, Matrix, Sequence Function.

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1. INTRODUCTION

Telecommunication is vital to FAA. The reason is more than just a means of communication, but it is because one of the communication terminals is usually airborne and unaccessible by cables. The growth of airplane users creates the same growing demand for telecommunications. All these point to a growing need for better spectrum utilization within the telecommunication spectrum.

A control tower or flight service station may communicate to more than one plane at a time. Therefore, in existing FAA facilities, multiple channels are provided at each location. But when frequency assignments are too close, cross modulation and inter-modulation can occur. On one hand, demand presents a need to squeeze in more channels into the existing spectrum. But on the other hand by having close frequency assignments it results in undesirable cross-modulations and inter-modulations. Resolutions of these problems require consideration of the following:

1. Selectivity of receivers.
2. Sensitivity of receivers.
3. Channel bandwidth of transmitters and power output of transmitters.
4. Sharp cutoff bandpass filters.
5. Using RF cancellation techniques.
6. Coding format.
7. Increase antenna isolation.
8. Other techniques: Time sharing, etc.

This report addresses itself to item number 7, improving antenna isolations.

2. SOME FAA COMMUNICATION ANTENNA SITUATIONS

2.1 Physical Situations

Communication antennas are sometimes clustered around a tower top or a roof top. For example, on top of a control tower, communication antennas share a space approximately 15 feet by 15 feet square. Thus in a typical situation, antennas are mounted less than 6 feet apart.

2.2 Existing Communication Antenna Problems

In sites visited, according to operators, equipment performances were quite satisfactory. Occasionally there were some loss of signal and fading. Cross talk is ever present in the communication system. A remedy existing now to solve the problem rests with the training of the operators. They are experienced enough to separate the voice of the other operator from the noise background. The adaptability of human being minimizes the problem. However, the problem is still there, and some of these problems can be solved by better equipment. Loss of signal and fading can be due to antenna problems. They could be due to poor antenna placement, such that other antennas or structures (like lightning rods) interferes with the antenna's radiation characteristics. It is seen that many of these problems can be solved or minimized by improving antenna isolations.

3. SOLUTIONS TO THE ANTENNA ISOLATION PROBLEM

When one looks at the problem of increasing isolation, and if one takes a systems approach one must first list the antenna parameters that will affect isolation. The major parameters involved here are:

1. Amplitude,
2. Phase, and
3. Polarization.

These are the electrical parameters to be considered and of course there are mechanical parameters like:

1. Size,
2. Rigidity,
3. Weatherability,
4. Reliability. And lastly but not least,
5. Compatibility with the rest of the communication system.

Consider first the three major electrical parameters, they are taken up one at a time in the following sections:

3.1 Amplitude Consideration

In this topic one takes into consideration antenna spacing and pattern shape. For FAA communication facilities, the desired pattern shape is generally omnidirectional in the azimuth plane. Therefore, isolation by taking advantage of pattern is usually not applicable. The only other thing is antenna spacing. To achieve 30 - 40 db isolation, the two antennas have to be spaced 80 or so feet apart. This is of course not possible if one confines the antennas to a common roof top or tower.

3.2 Phase Considerations

Existing ideas involve the use of a feedback loop. If the feed back contains frequencies and phases that are different from the desired signal, these extraneous signals can be suppressed by phase opposition. It is difficult to determine and use the coupling characteristics of all the antennas involved in order to create proper phase cancellation, without deteriation of the desired signal.

Another method without using the feedback loop

ends up as a circular array system. The system is discussed in section 4.

3.3 Polarization Considerations

Isolation by polarization diversity is very effective. But for air-ground communication in use by FAA, the types of polarizations applicable are limited. They are limited to:

1. Vertical Polarization,
2. Right Handed Circular Polarization,
3. Left Handed Circular Polarization.

Polarization diversity using a vertically polarized antenna and a horizontally polarized antenna can not be used in FAA facilities. Another way is to use circular polarizations of different rotations. It has been shown that relatively high isolation can be achieved by using such an arrangement. However, this technique only provides one additional isolated antenna.

3.4 Vertical Stacking

By vertically stacking two vertically polarized antennas, in theory, high isolation can be achieved. But in practice, it is impossible to achieve high isolation if existing antennas are used, because the feeding cable of the antenna on top will interfere with the radiation characteristics of the lower antenna. Furthermore, isolation value is very sensitive to the alignment of the two antennas. However this does not exclude the possibility of designing two antennas coaxially mounted. Such that the feed cable of the antenna on top can go straight through the body of the lower antenna.

3.5 Some Mechanical Considerations

Some desirable features that may be incorporated into a design are that the antennas should be constructed as a module so that stacking can be done to increase the number of units. Some kind of weather protection shield should be provided in a form of radomes to minimize weather interference, and also to hold antenna tolerances. The construction of supporting structures should be designed so as to minimize interference with antennas' electrical characteristics. For example, tower railings can create a ground plane effect. And

also, lightning rods can shadow the antenna radiation. It is impossible to design an effective antenna system without taking into consideration of supporting structures.

4. CIRCULAR ARRAY CONCEPT

It is a known fact that if a circular array is fed with a progressive phase, it would create a null at the array center. If an antenna is placed at the null point, then perfect isolation can be achieved. This is the concept of using circular array to achieve antenna isolation. Further, if the array is fed by a Shelton-Butler matrix then multiple beams can be formed. In this case, each desired beam is doughnut shaped and is formed by a different phase progression.

4.1 Advantages of Using a Circular Array

Because of the possibility of simultaneous beam forming, many isolated channels equivalent to many isolated antennas can be formed simultaneously. Another advantage is the small size, because a circular array forms the isolated channels within itself, the size is thus limited to the size of the circular array itself. High degree of isolation is achieved through precision and not by size.

The concept of the phased circular array is the mixture of amplitude consideration and phase consideration, in that all the array elements must be fed with equal amplitude and the phase is progressive. If the amplitudes are not equal, then the resultant null would not be a deep one. Using this concept, polarization diversity does not have to enter into the picture. However, polarization may be used to further the advantage gained by using the array.

The array analyzed in this report employs vertically polarized elements. The concept can be extended to circularly polarized antennas.

In order to achieve perfect isolation the element to be isolated from the array must be placed perfectly at the null point. But in fact that is not possible, therefore a sizable portion of this report is devoted to finding the sensitivity of position error on the magnitude of isolation. In the next section the theoretical formulation of the sensitivity study will be presented and its' results discussed.

5. THEORETICAL STUDIES

The basic technique employed in this research is the moments method. The formula used is the Hallen's Integral Equation. The array configuration analyzed is shown in Figure 1. A computer program is written to analyze such an array. In the analysis it is assumed that there are four elements equally spaced around a circle and a fifth element placed near the center of the circle. The location of the center element is varied to study its effect of radiation patterns, current distributions, and isolation. The computer program itself however is quite general. It can handle array elements placed in any location and they do not have to be placed on a circle.

The center element is loaded with a fixed resistance. In the program the value of the resistance can be varied, but for the purpose of this study and for uniformity of results in order to facilitate comparisons, the resistance value is fixed at 100 ohms.

The detailed discussion of the Moments Method, the formulation of Hallen's Integral can be found in Harrington [1] and Kraus [2]. Assumptions employed in writing the computer program and the program listing are included in Appendix A.

The dipole elements are assumed to be half wavelength long center fed and symmetrical. The feed points are all on the same plane. The feed voltages are labeled V_1, V_2 to V_5 as shown in Figure 1. A unique technique called sequence function method is employed while calculating the input impedances and the technique is described in Appendix B. The output of the program shown in Appendix A includes the generalized admittance matrix, which leads to current distributions and input impedances of the antennas.

A separate program using a different approach also calculates the current distribution and impedances. The program is listed in Appendix C.

A third program which uses the output of the second program to calculate input impedances and current distributions along with a fourth program which calculates radiation patterns and phases, as well as plotting them, are also included in Appendix C.

A fifth program which uses the current distribution

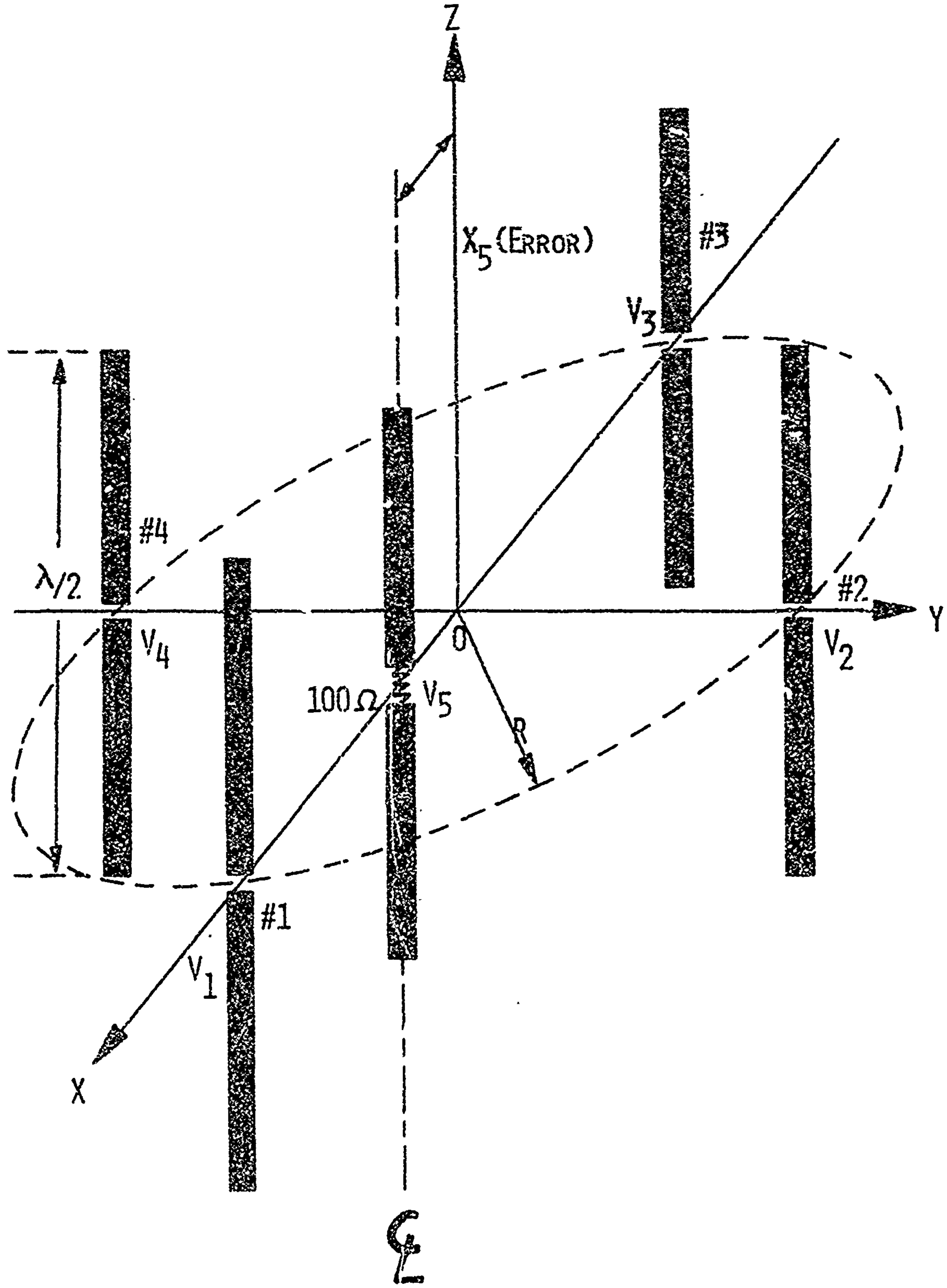


Figure 1 : A Four-Element Circular Array, Progressively Phased. A Fifth Element is Placed Near Array Center, Where the Null is.

data to calculate isolation data is appended in Appendix C. A sixth program which scales and superpositions the phase sequences to obtain final field patterns is also appended in Appendix C. Theoretical results have been obtained using the four phase progressions shown in Table 1. They are discussed in following sections. It is assumed that the center element is terminated with a 100 ohm load.

5.1 Current Distributions

After obtaining the generalized admittance matrix, the first information that is measurable is the current distribution along the length of the dipole antennas. The case where the array diameter is 0.3 wavelength and the dipole cross-section radius 0.025 wavelength is considered first. Figures 2 through 4 show current distributions for four different phase sequences. It is seen in Figure 2 that the current of the center element for the zero degree progression is very large. That is because all the dipole elements are in phase and their powers reinforce each other at the location of the center element. However, Figure 3 shows the current distribution of the case of plus and minus 90 degrees. It is seen there that the center element current distribution is very small and that is the basis of high isolation when the phased circular array concept is used. Figure 4 shows the similar current distribution when the phase progression is increased to 180 degrees. The current distribution on the center element is again very small.

Results of other array configurations are shown in Appendix D.

5.2 Short-Circuited Admittances

Tables 2 through 4 show the short-circuited admittances of the above-mentioned array. They are provided as an intermediate step for other calculations. Namely, input impedance and admittance calculations and load current calculations.

Results of other array configurations are shown in Appendix D.

5.3 Input Impedance

PHASE PROGRESSIONS	0°	90°	-90°	180°
V_1	$1\angle 0^\circ$	$1\angle 0^\circ$	$1\angle 0^\circ$	$1\angle 0^\circ$
V_2	$1\angle 0^\circ$	$1\angle 90^\circ$	$1\angle -90^\circ$	$1\angle 180^\circ$
V_3	$1\angle 0^\circ$	$1\angle 180^\circ$	$1\angle -180^\circ$	$1\angle 0^\circ$
V_4	$1\angle 0^\circ$	$1\angle 270^\circ$	$1\angle -270^\circ$	$1\angle 180^\circ$

Table 1 : Voltage and Phases of Array Elements to Create Various Modes.

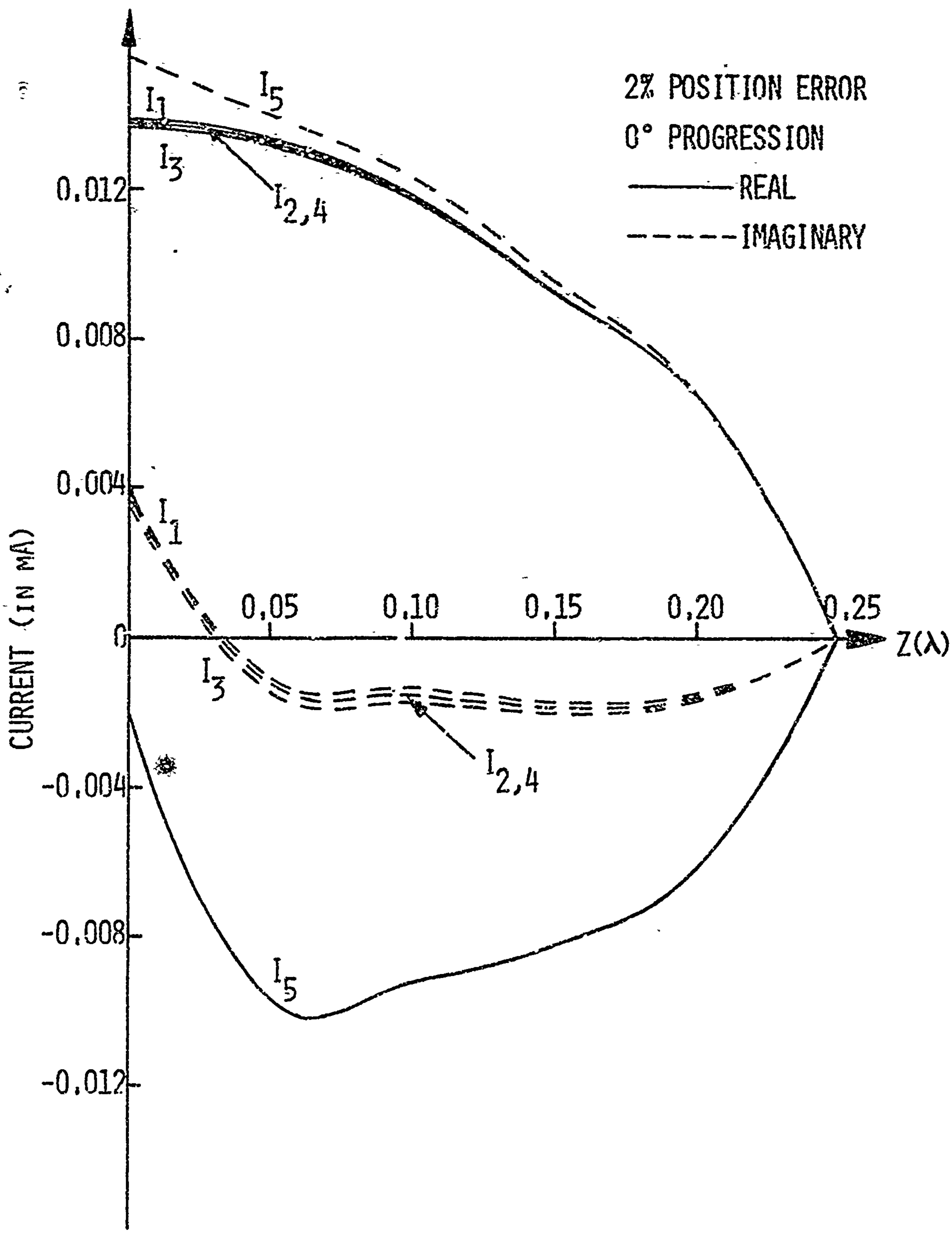


Figure 2 : Current Distribution for the 0° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 2%.

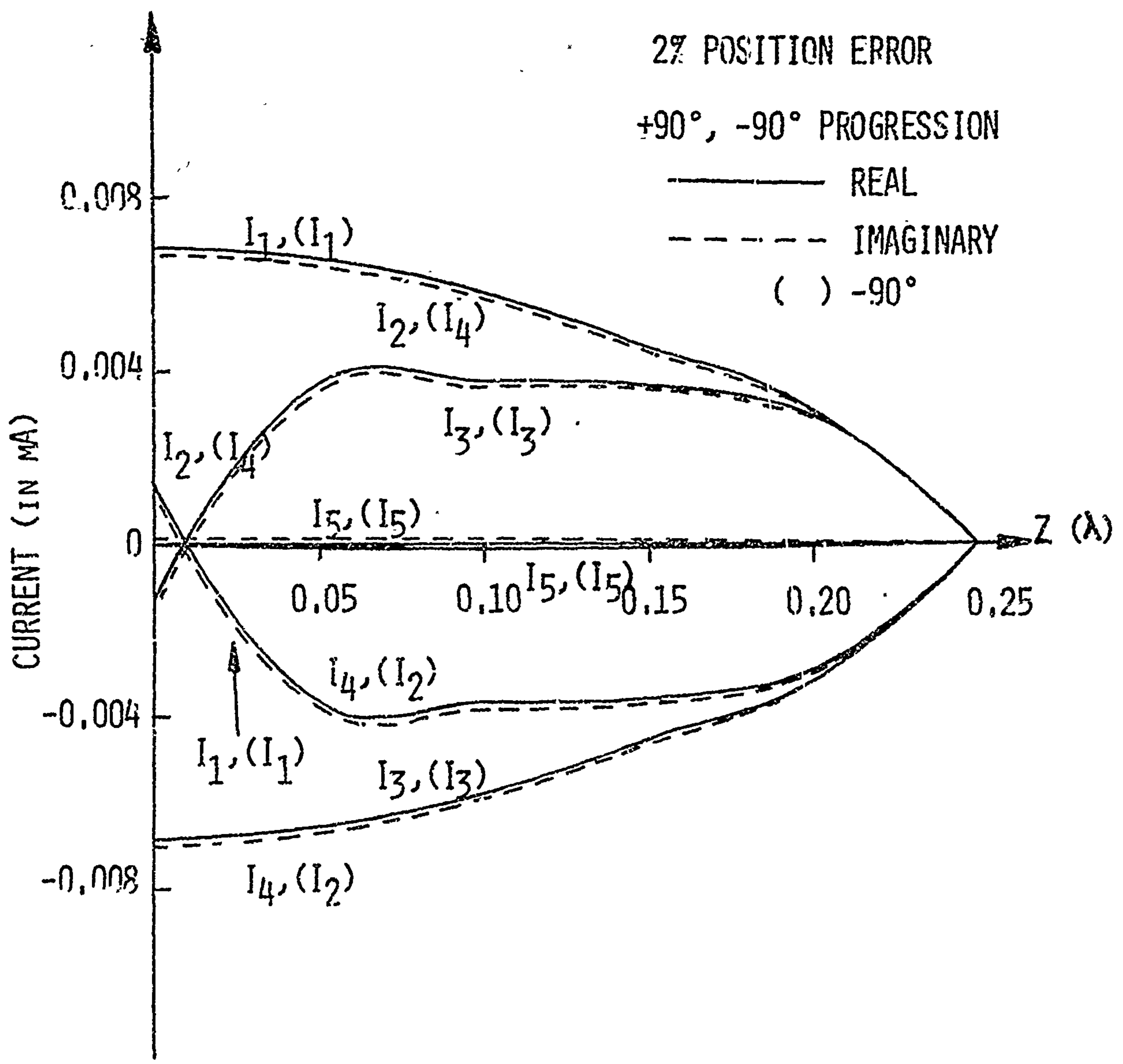


Figure 3 : Current Distribution for the +90° and -90° Phase Progressions. Radius of the Array (R) is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 2%.

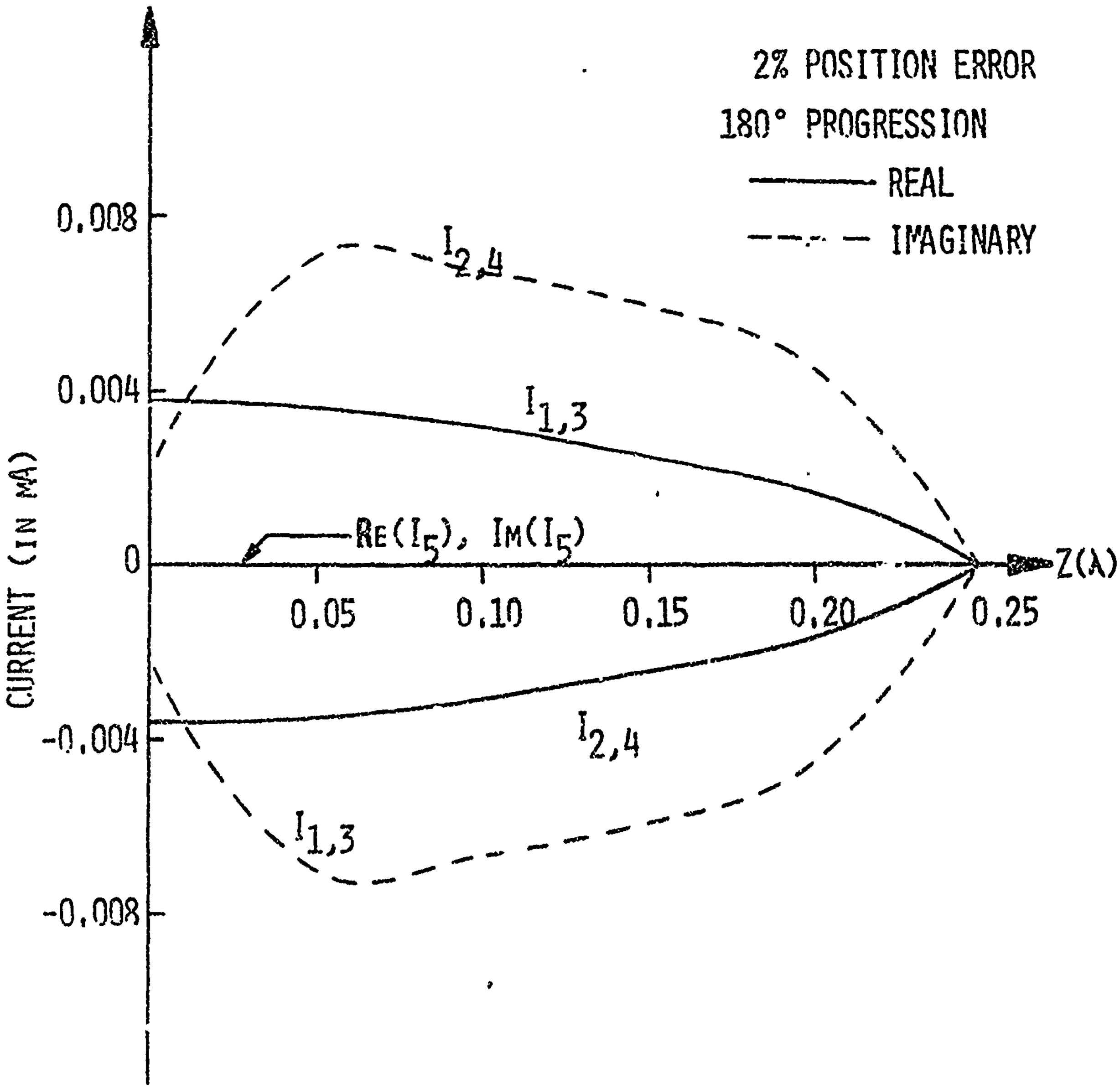


Figure 4 : Current Distribution for the 180° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 2%.

A = 0.0250
R = 0.3000
X_5= 0.0015
Y_5= 0.0000

ERROR = 0.5%

Y_{11} = 0.61661E-02+j0.99869E-03	Y_{12} = 0.90292E-03+j0.12915E-02
Y_{13} =-0.67795E-03-j0.42672E-03	Y_{14} = 0.90292E-03+j0.12915E-02
Y_{15} = 0.74757E-04+j0.41536E-02	Y_{21} = 0.90292E-03+j0.12915E-02
Y_{22} = 0.61705E-02+j0.10275E-02	Y_{23} = 0.90732E-03+j0.13201E-02
Y_{24} =-0.67797E-03-j0.42664E-03	Y_{25} = 0.73529E-04+j0.41301E-02
Y_{31} =-0.67795E-03-j0.42672E-03	Y_{32} = 0.90732E-03+j0.13201E-02
Y_{33} = 0.61749E-02+j0.10559E-02	Y_{34} = 0.90732E-03+j0.13201E-02
Y_{35} = 0.72273E-04+j0.41070E-02	Y_{41} = 0.90292E-03+j0.12915E-02
Y_{42} =-0.67797E-03-j0.42664E-03	Y_{43} = 0.90732E-03+j0.13201E-02
Y_{44} = 0.61705E-02+j0.10275E-02	Y_{45} = 0.73529E-04+j0.41301E-02
Y_{51} = 0.74758E-04+j0.41536E-02	Y_{52} = 0.73532E-04+j0.41301E-02
Y_{53} = 0.72276E-04+j0.41070E-02	Y_{54} = 0.73532E-04+j0.41301E-02
Y_{55} = 0.45548E-03-j0.15182E-02	

Z_{11} = 0.12410E+03+j0.91576E+01	Z_{12} =-0.50212E+02-j0.33150E+02
Z_{13} =-0.15344E+02+j0.38775E+02	Z_{14} =-0.50212E+02-j0.33150E+02
Z_{15} = 0.79162E+01-j0.53309E+02	Z_{21} =-0.50212E+02-j0.33150E+02
Z_{22} = 0.12437E+03+j0.91088E+01	Z_{23} =-0.49943E+02-j0.33201E+02
Z_{24} =-0.15343E+02+j0.38776E+02	Z_{25} = 0.73753E+01-j0.52885E+02
Z_{31} =-0.15344E+02+j0.38775E+02	Z_{32} =-0.49943E+02-j0.33201E+02
Z_{33} = 0.12464E+03+j0.90552E+01	Z_{34} =-0.49943E+02-j0.33201E+02
Z_{35} = 0.68464E+01-j0.52459E+02	Z_{41} =-0.50212E+02-j0.33150E+02
Z_{42} =-0.15343E+02+j0.38776E+02	Z_{43} =-0.49943E+02-j0.33201E+02
Z_{44} = 0.12437E+03+j0.91089E+01	Z_{45} = 0.73753E+01-j0.52885E+02
Z_{51} = 0.79162E+01-j0.53309E+02	Z_{52} = 0.73753E+01-j0.52885E+02
Z_{53} = 0.68463E+01-j0.52459E+02	Z_{54} = 0.73753E+01-j0.52885E+02
Z_{55} = 0.86780E+02+j0.55720E+02	

Table 2: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

$A = 0.0250$
$R = 0.3000$
$X_5 = 0.0060$
$Y_5 = 0.0000$

ERROR = 2.0%

Y_{11} = 0.61527E-02+j0.91060E-03	Y_{12} = 0.89628E-03+j0.12488E-02
Y_{13} =-0.67772E-03-j0.42652E-03	Y_{14} = 0.89628E-03+j0.12488E-02
Y_{15} = 0.78434E-04+j0.42251E-02	Y_{21} = 0.89628E-03+j0.12488E-02
Y_{22} = 0.61705E-02+j0.10289E-02	Y_{23} = 0.91387E-03+j0.13632E-02
Y_{24} =-0.67805E-03-j0.42529E-03	Y_{25} = 0.73740E-04+j0.41299E-02
Y_{31} =-0.67772E-03-j0.42652E-03	Y_{32} = 0.91387E-03+j0.13632E-02
Y_{33} = 0.61879E-02+j0.11396E-02	Y_{34} = 0.91387E-03+j0.13632E-02
Y_{35} = 0.68518E-04+j0.40388E-02	Y_{41} = 0.89628E-03+j0.12488E-02
Y_{42} =-0.67805E-03-j0.42529E-03	Y_{43} = 0.91387E-03+j0.13632E-02
Y_{44} = 0.61705E-02+j0.10289E-02	Y_{45} = 0.73740E-04+j0.41299E-02
Y_{51} = 0.78433E-04+j0.42251E-02	Y_{52} = 0.73740E-04+j0.4129[illegible]E-02
Y_{53} = 0.68520E-04+j0.40388E-02	Y_{54} = 0.73740E-04+j0.4129[illegible]
Y_{55} = 0.45556E-03-j0.15211E-02	

Z_{11} = 0.12327E+03+j0.92764E+01	Z_{12} =-0.50609E+02-j0.33080E+02
Z_{13} =-0.15330E+02+j0.38771E+02	Z_{14} =-0.50609E+02-j0.33080E+02
Z_{15} = 0.95590E+01-j0.54543E+02	Z_{21} =-0.50609E+02-j0.33080E+02
Z_{22} = 0.12439E+03+j0.91139E+01	Z_{23} =-0.49535E+02-j0.33285E+02
Z_{24} =-0.15324E+02+j0.38781E+02	Z_{25} = 0.73234E+01-j0.52867E+02
Z_{31} =-0.15330E+02+j0.38771E+02	Z_{32} =-0.49535E+02-j0.33285E+02
Z_{33} = 0.12541E+03+j0.88690E+01	Z_{34} =-0.49535E+02-j0.33285E+02
Z_{35} = 0.52809E+01-j0.51144E+02	Z_{41} =-0.50609E+02-j0.33080E+02
Z_{42} = 0.15324E+02+j0.38781E+02	Z_{43} =-0.49535E+02-j0.33285E+02
Z_{44} = 0.12439E+03+j0.91139E+01	Z_{45} = 0.73234E+01-j0.52865E+02
Z_{51} = 0.95590E+01-j0.54543E+02	Z_{52} = 0.73234E+01-j0.52865E+02
Z_{53} = 0.52809E+01-j0.51144E+02	Z_{54} = 0.73234E+01-j0.52865E+02
Z_{55} = 0.86762E+02+j0.55632E+02	

Table 3: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.3000
X_5 = 0.0300
Y_5 = 0.0000

ERROR = 10.0%

Y_{11} = 0.60766E-02+j0.39141E-03	Y_{12} = 0.86030E-03+j0.10255E-02
Y_{13} =-0.67189E-03-j0.42157E-03	Y_{14} = 0.86030E-03+j0.10255E-02
Y_{15} = 0.96820E-04+j0.46448E-02	Y_{21} = 0.86030E-03+j0.10255E-02
Y_{22} = 0.61686E-02+j0.10635E-02	Y_{23} = 0.94770E-03+j0.15956E-02
Y_{24} =-0.67997E-03-j0.39072E-03	Y_{25} = 0.79073E-04+j0.41244E-02
Y_{31} =-0.67189E-03-j0.42157E-03	Y_{32} = 0.94770E-03+j0.15956E-02
Y_{33} = 0.62516E-02+j0.15457E-02	Y_{34} = 0.94770E-03+j0.15956E-02
Y_{35} = 0.48121E-04+j0.37055E-02	Y_{41} = 0.86030E-03+j0.10255E-02
Y_{42} =-0.67997E-03-j0.39072E-03	Y_{43} = 0.94770E-03+j0.15956E-02
Y_{44} = 0.61686E-02+j0.10635E-02	Y_{45} = 0.79073E-04+j0.41244E-02
Y_{51} = 0.96819E-04+j0.46448E-02	Y_{52} = 0.79073E-04+j0.41244E-02
Y_{53} = 0.48122E-04+j0.37055E-02	Y_{54} = 0.79073E-04+j0.41244E-02
Y_{55} = 0.45776E-03-j0.15965E-02	

Z_{11} = 0.11829E+03+j0.90971E+01	Z_{12} =-0.52565E+02-j0.32794E+02
Z_{13} =-0.14988E+02+j0.38652E+02	Z_{14} =-0.52565E+02-j0.32794E+02
Z_{15} = 0.18976E+02-j0.60223E+02	Z_{21} =-0.52565E+02-j0.32794E+02
Z_{22} = 0.12490E+03+j0.92330E+01	Z_{23} =-0.47305E+02-j0.33921E+02
Z_{24} =-0.14822E+02+j0.38900E+02	Z_{25} = 0.60056E+01-j0.52355E+02
Z_{31} =-0.14988E+02+j0.38652E+02	Z_{32} =-0.47305E+02-j0.33921E+02
Z_{33} = 0.12886E+03+j0.73442E+01	Z_{34} =-0.47305E+02-j0.33921E+02
Z_{35} =-0.21861E+01-j0.43429E+02	Z_{41} =-0.52565E+02-j0.32794E+02
Z_{42} =-0.14822E+02+j0.38900E+02	Z_{43} =-0.47305E+02-j0.33921E+02
Z_{44} = 0.12490E+03+j0.92330E+01	Z_{45} = 0.60056E+02-j0.52355E+02
Z_{51} = 0.18976E+02-j0.60223E+02	Z_{52} = 0.60056E+01-j0.52355E+02
Z_{53} =-0.21862E+01-j0.43429E+02	Z_{54} = 0.60056E+02-j0.52355E+02
Z_{55} = 0.86336E+02+j0.53419E+02	

Table 4: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

It is desirable to know how sensitive the array's input impedance is to the center element's position. This gives the information on how the feeding network must be adjusted to achieve good power transfer.

For the 0° progression, and when array radius is 0.3 wavelength, it is seen in Figure 5 that the real part of the impedance (resistance) does not change with position error, and some change in the imaginary part (reactance) as shown in Figure 6. Ports 1 and 3 are affected, because the center element is displaced towards element 1 and away from element 3, whereas the distance from elements 2 and 4 are practically unchanged.

When the array radius is increased to 0.5 wavelength, results are shown in Figures 7 and 8. Resistances of ports 1 and 3 are seen to change with position error.

Returning to the case where array radius is 0.3 wavelength, but changing the phase progression to 90°, it was found that all ports of the circular array are insensitive to position error. Results are shown in Figures 9 and 10. However, it is seen in the figures that the input impedance of the center element (port 5) changes drastically with position error, and seems to approach infinity at zero position error. That means for a fixed load at that port, impedance mismatch is greater when the center element is closer to array center. In the same manner, isolation between the element and the array goes up with decreasing error.

5.4 Radiation Pattern

Calculations were made of the circular array where the center element was loaded with 100 ohms. Results are plotted in the following figures.

Figures 11 through 13 are for an array with 0.3 wavelength radius at increasing position errors. It is seen that pattern minima of the 0°, 90°, 180° phase progressions are -1.2, -3.2 and $-\infty$ db respectively. Patterns of the +90° and -90° progressions are identical. With increasing error, pattern distortions are seen to be within 1 db.

Figures 14 through 16 show radiation patterns of the array when array radius is increased to 0.5 wave-

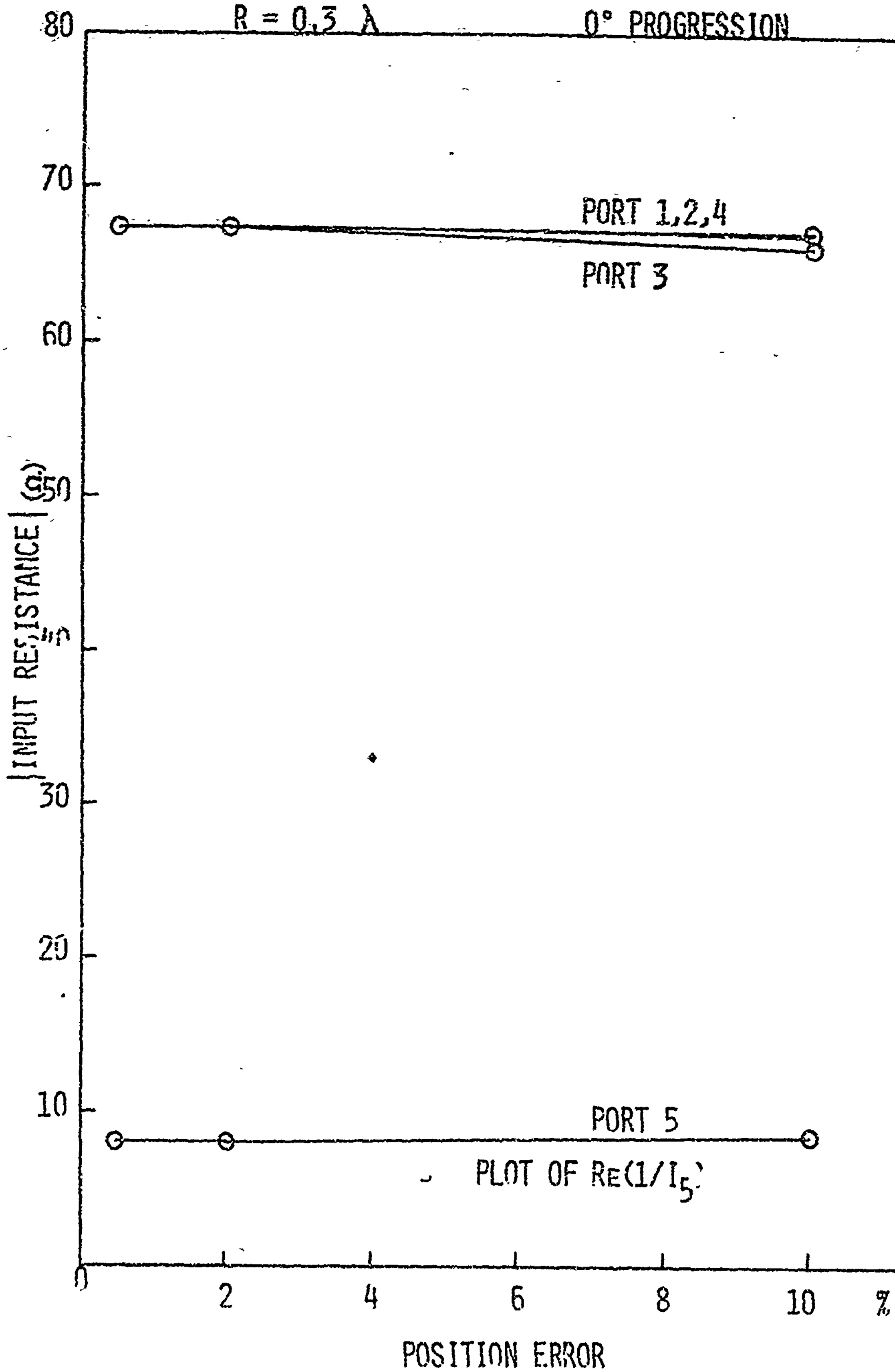


Figure 5 : Magnitude of Input Resistance as a Function of Position Error for the 0° Progression, 0.3 Wavelength Radius.

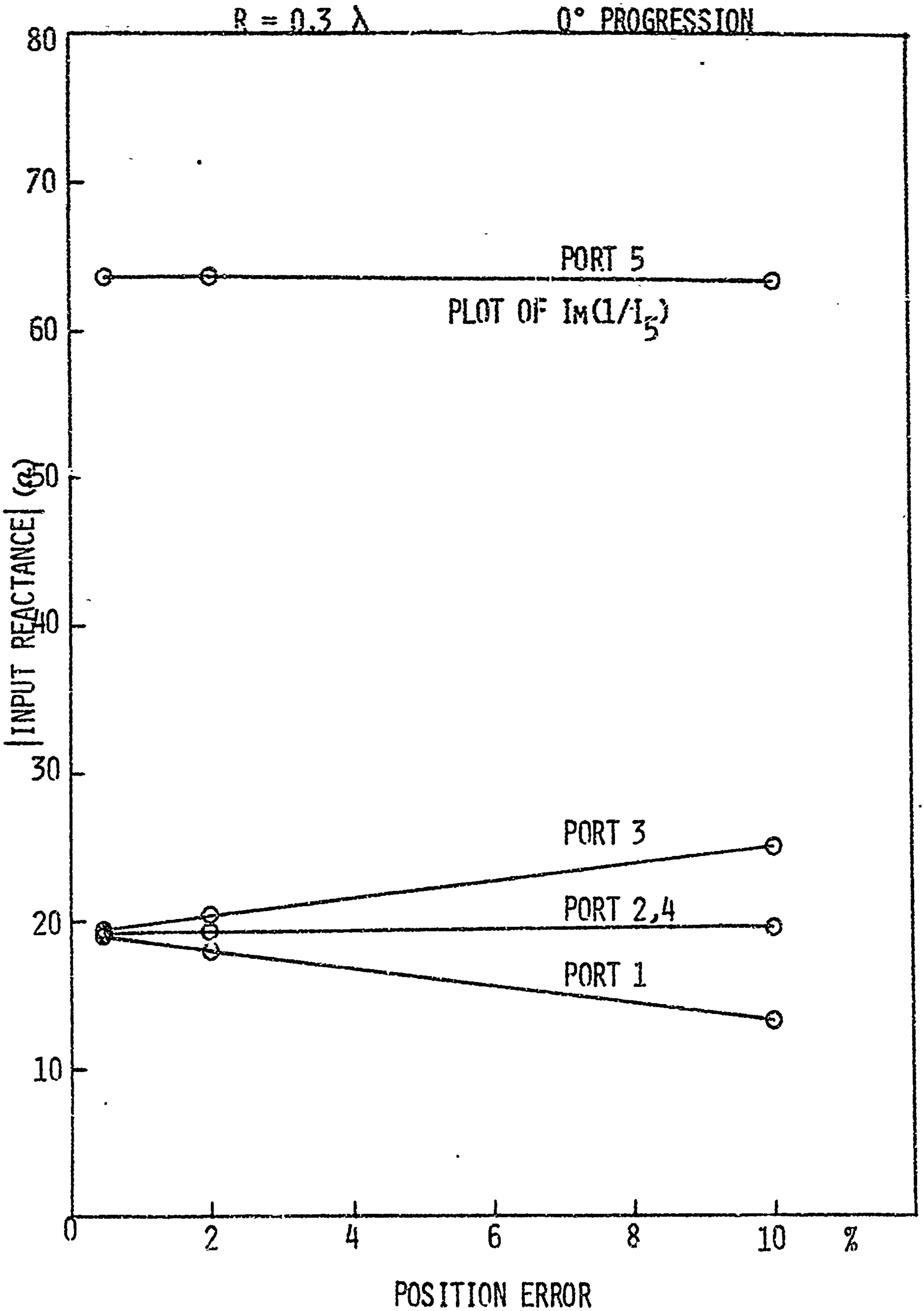


Figure 6 : Magnitude of Input Reactance of Array in Figure 5.

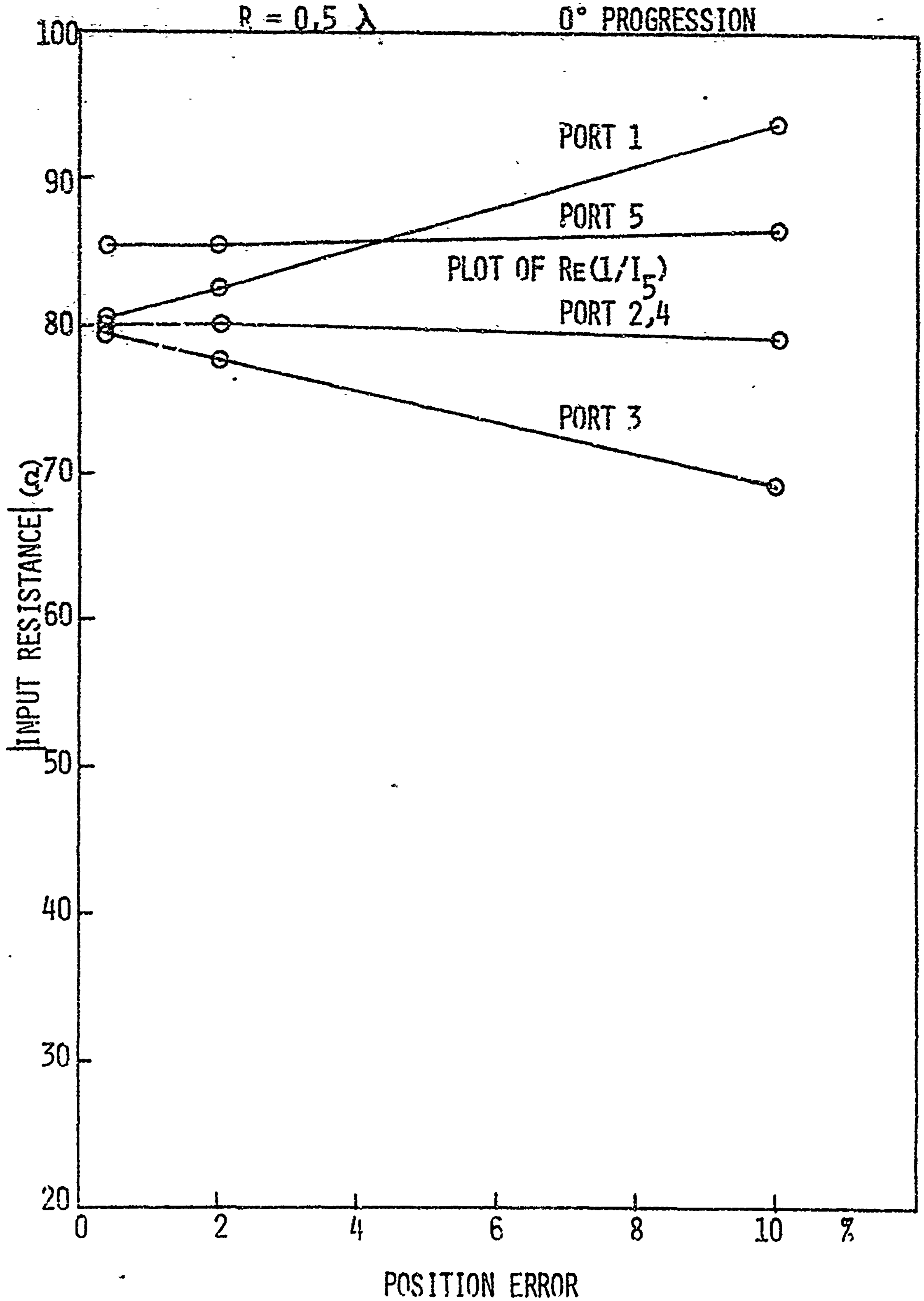


Figure 7 : Magnitude of Input Resistance as a function of Position Error for the 0° Progression, 0.5 Wavelength Radius.

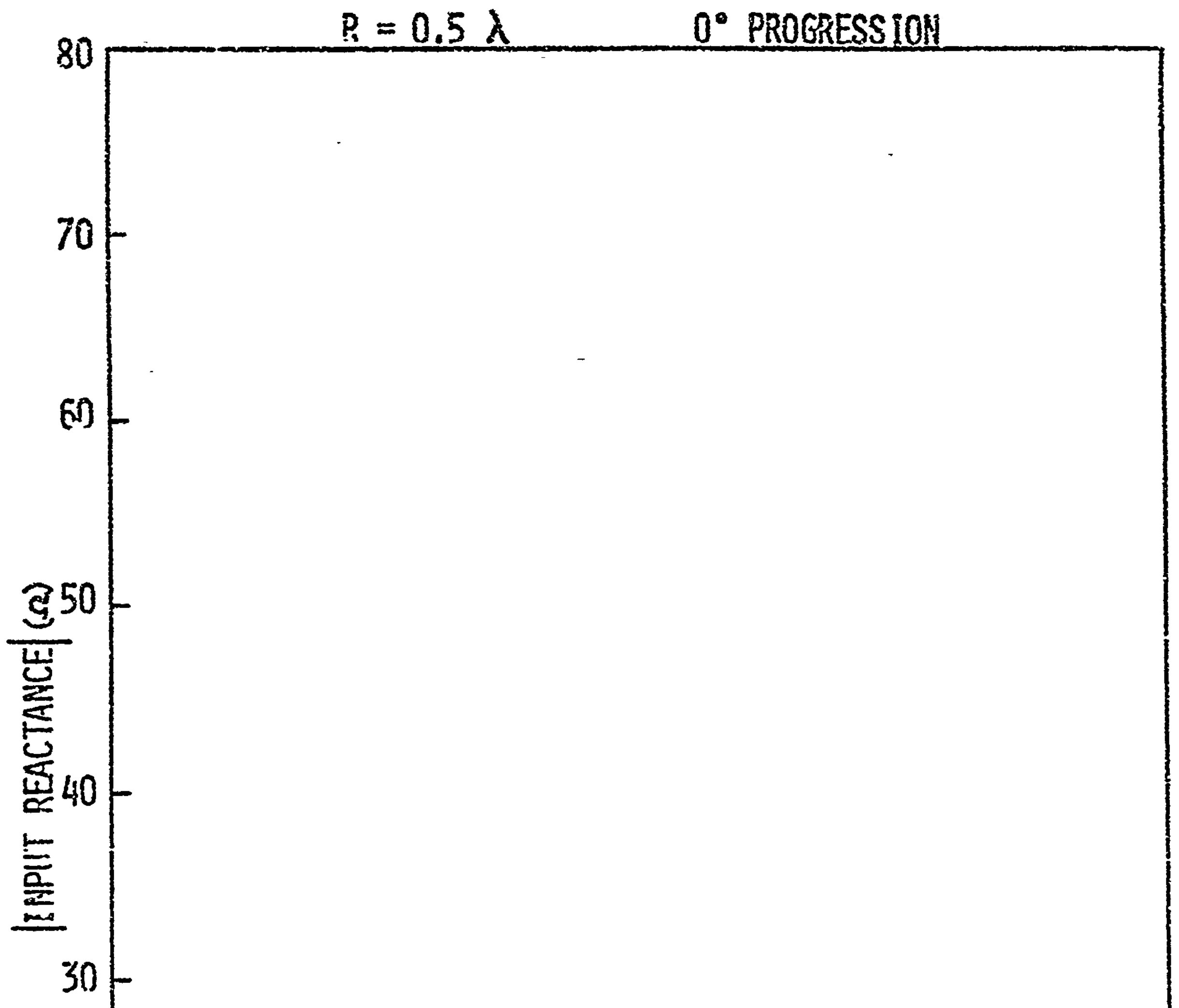


Figure 8 : Magnitude of Input Reactance of Array in Figure 7.

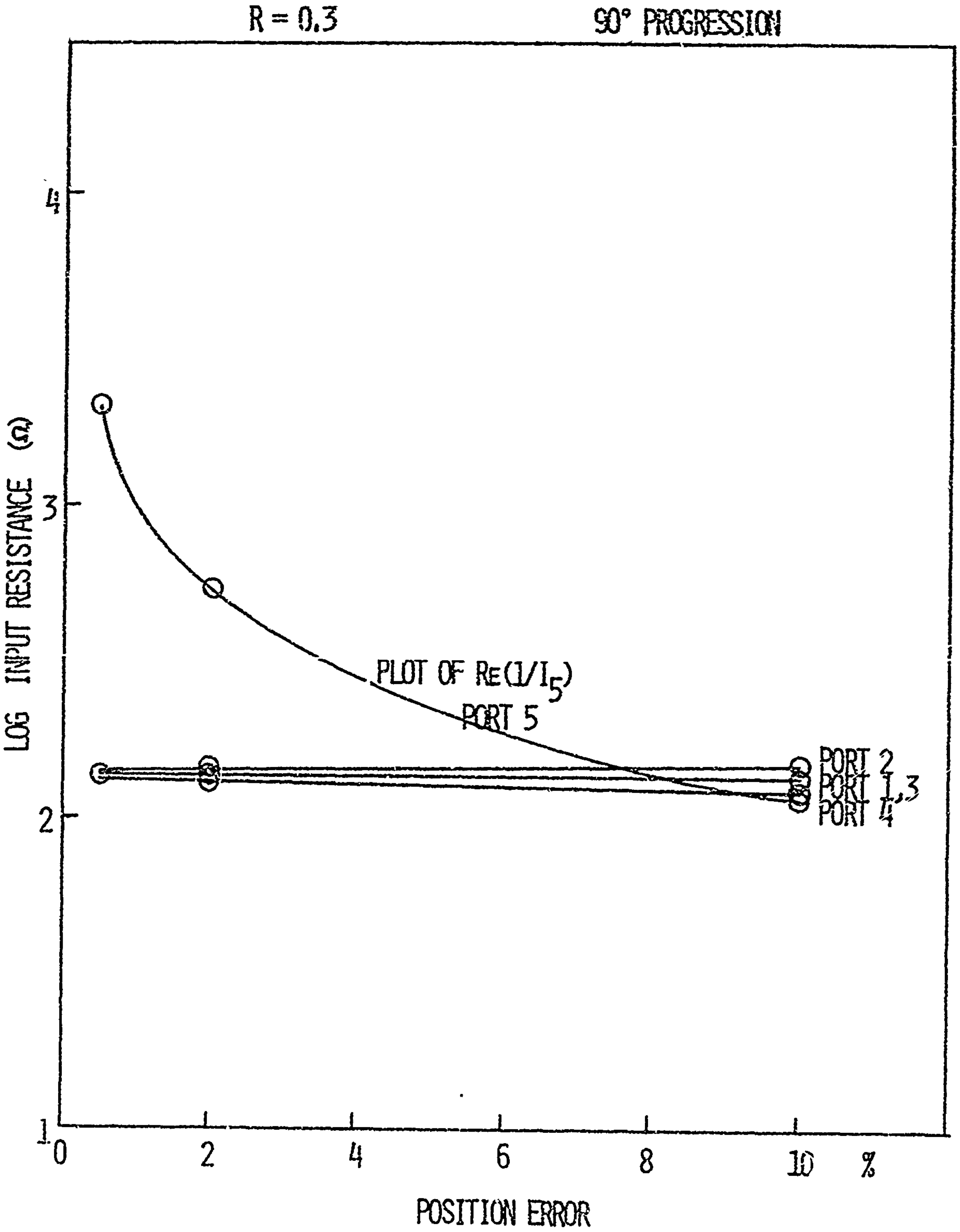


Figure 9 : Logarithmic Input Resistance as a Function of Position for The 90° Progression 0.3 Wavelength Radius.

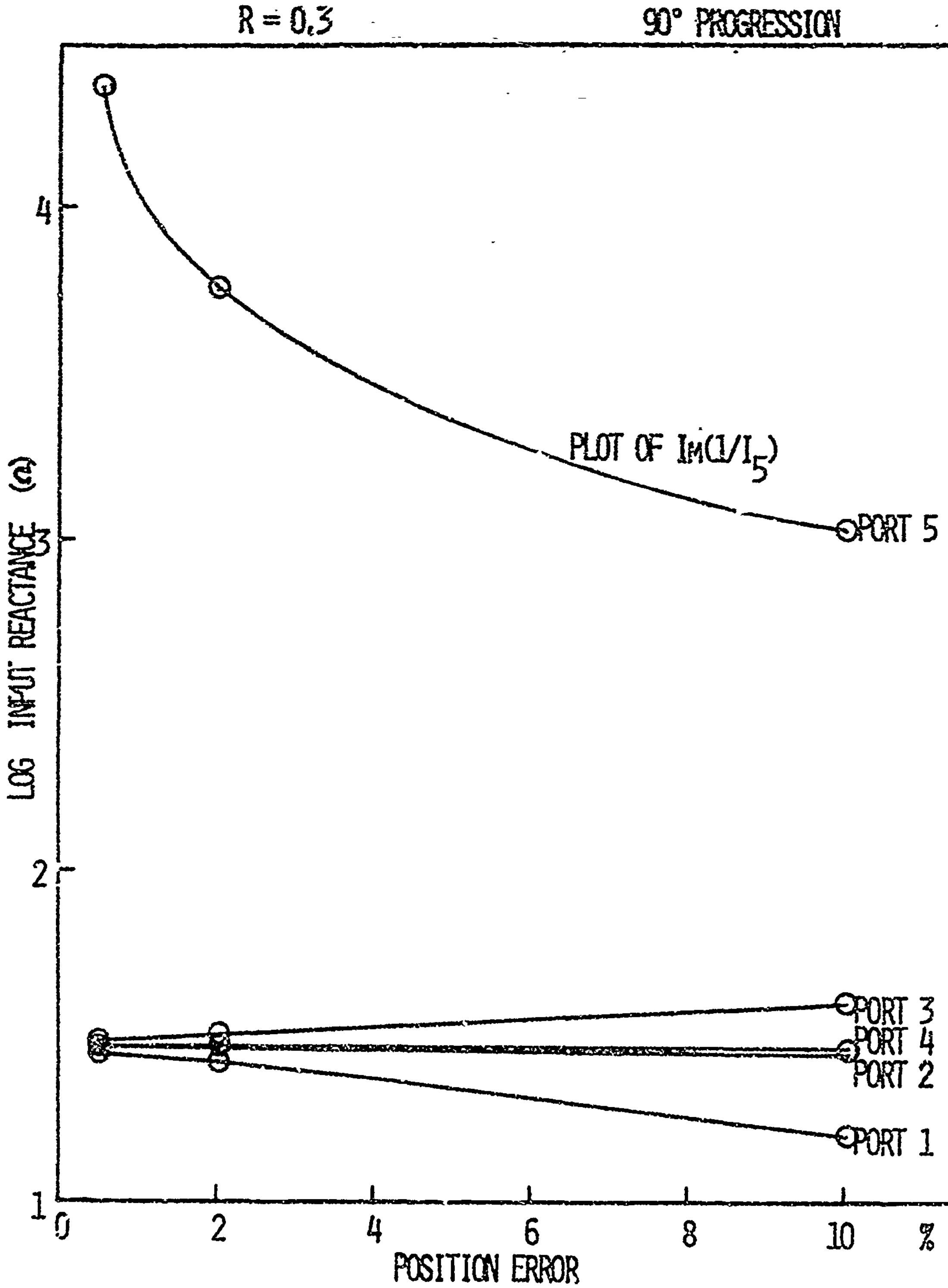


Figure 10 : Logarithmic Input Reactance of Array in Figure 9.

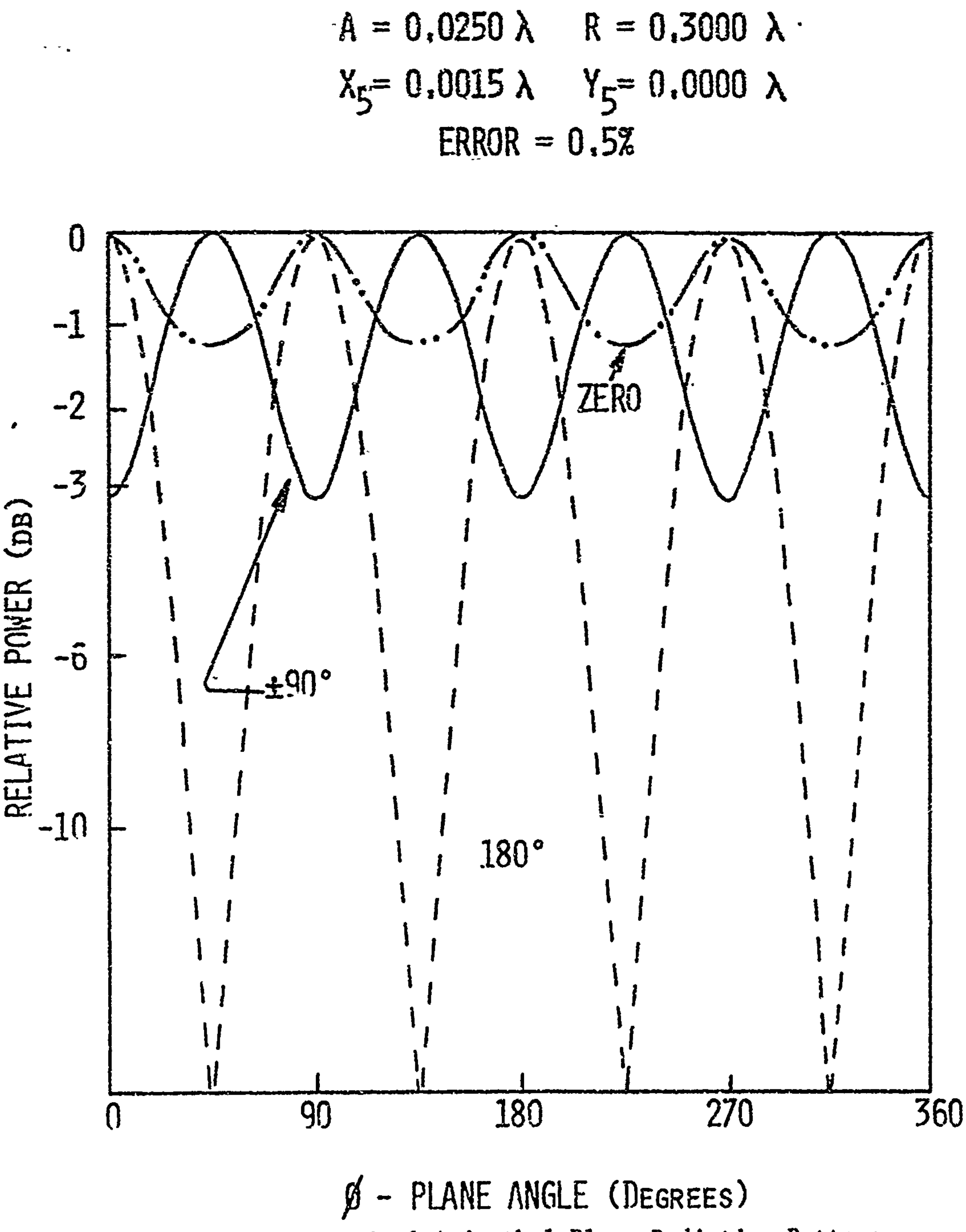


Figure 11 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

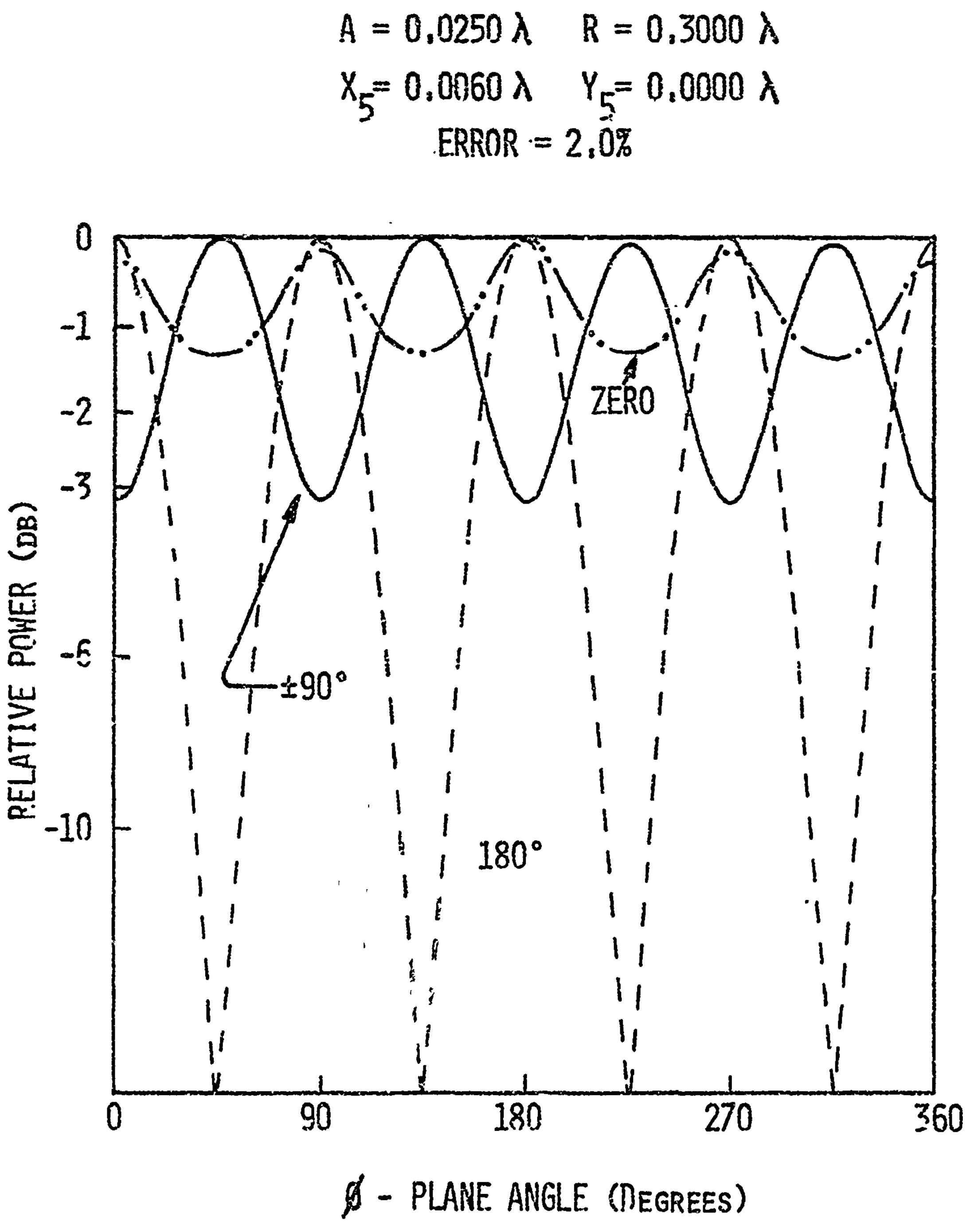


Figure 12 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

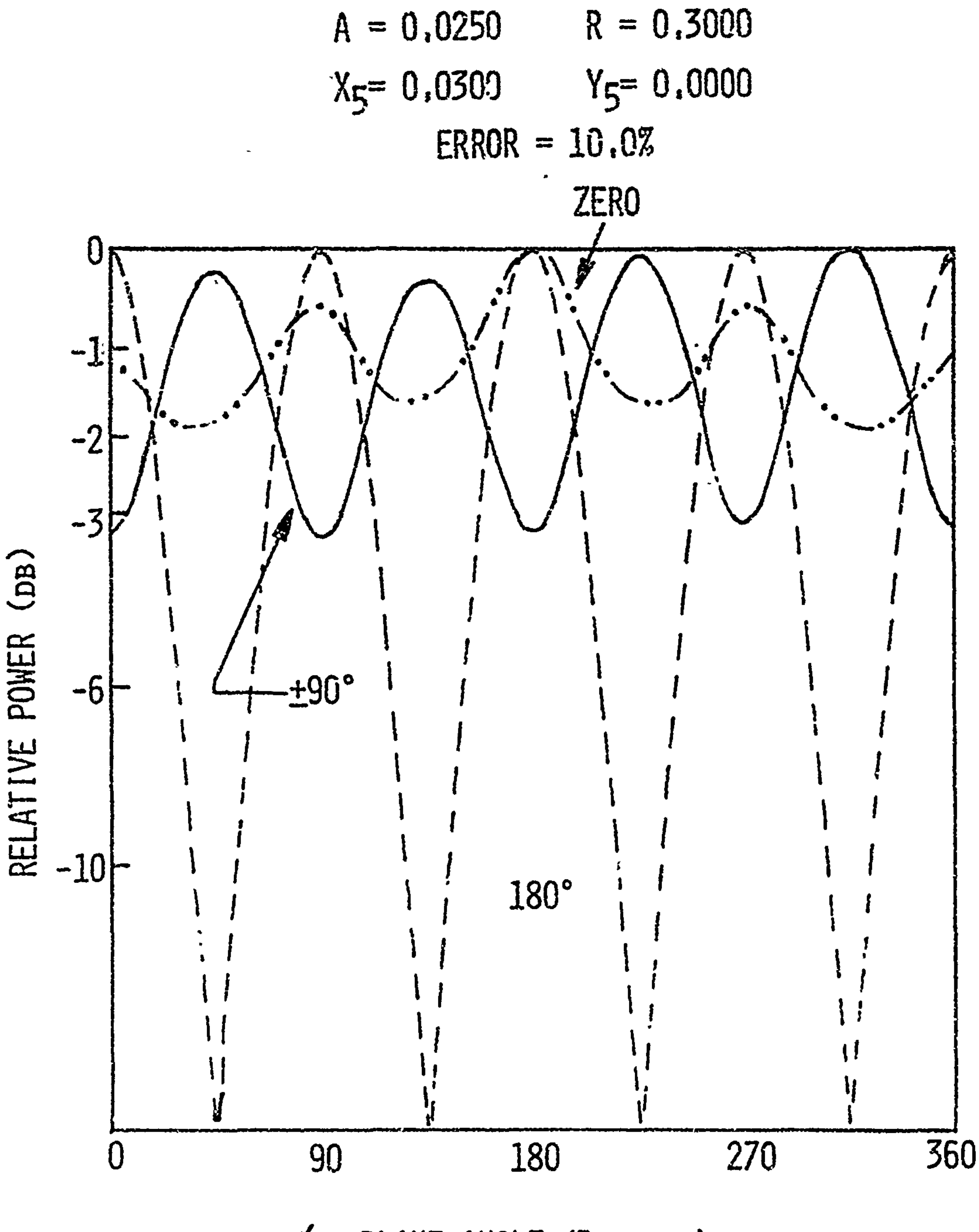


Figure 13 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

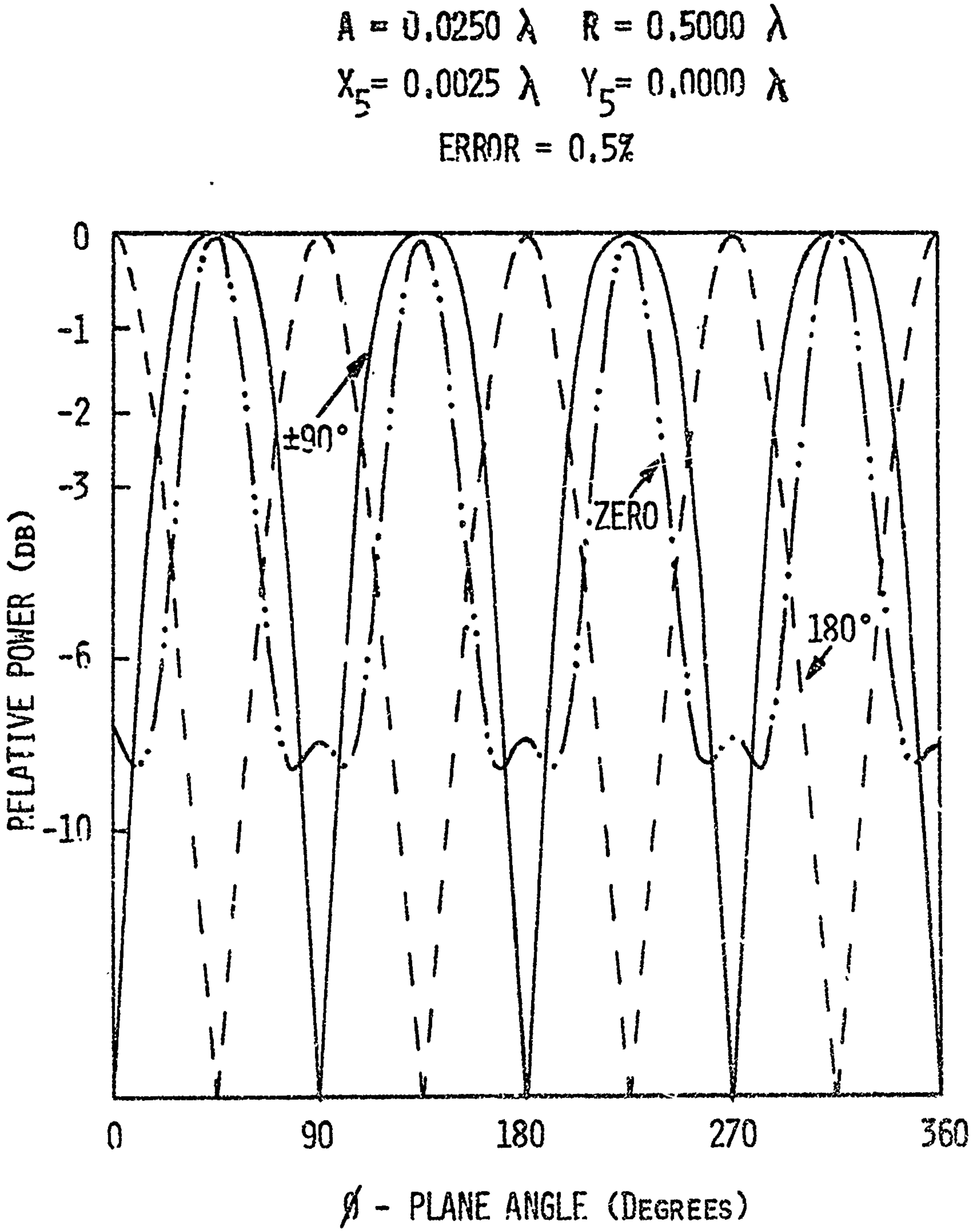


Figure 14 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

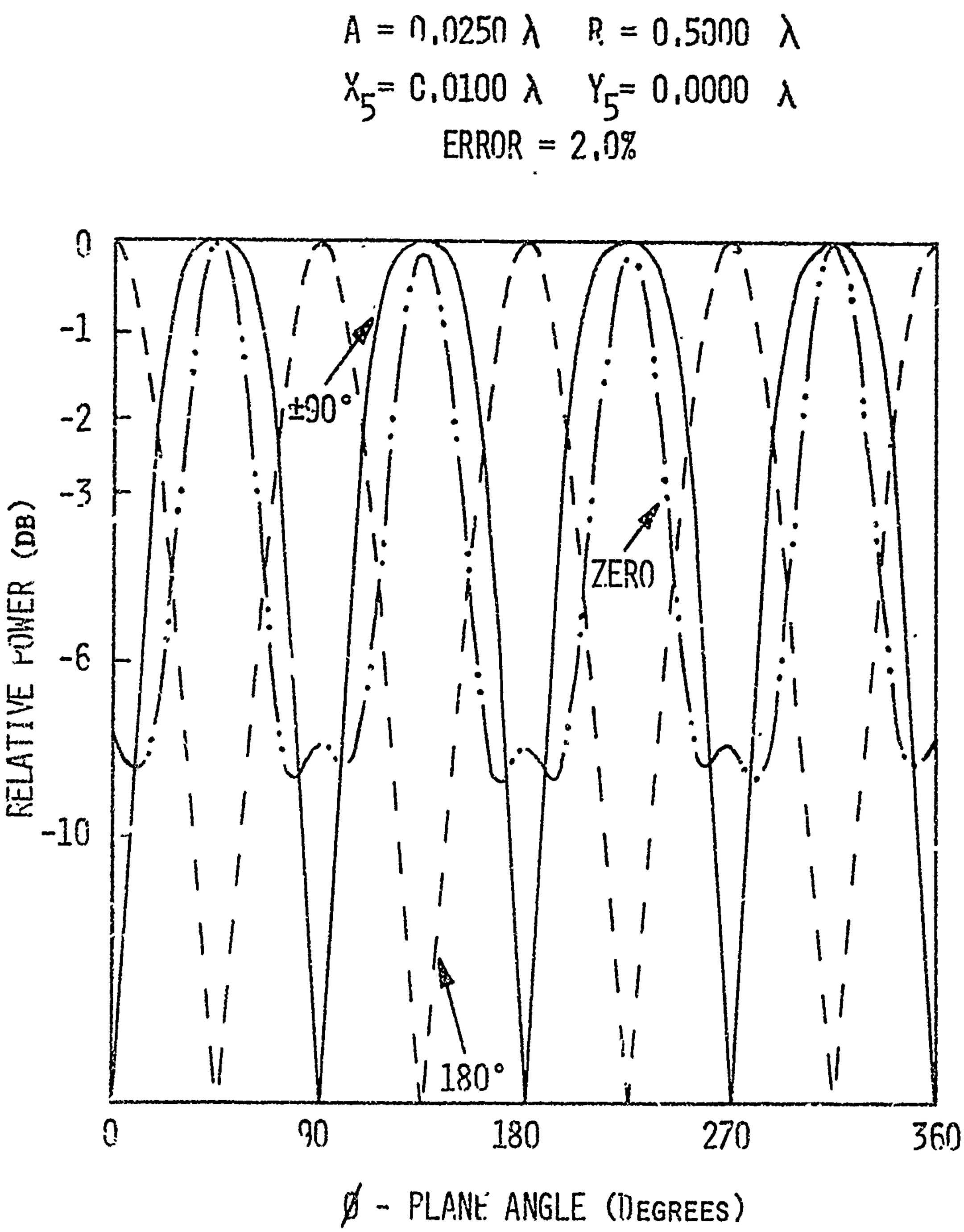


Figure 15 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

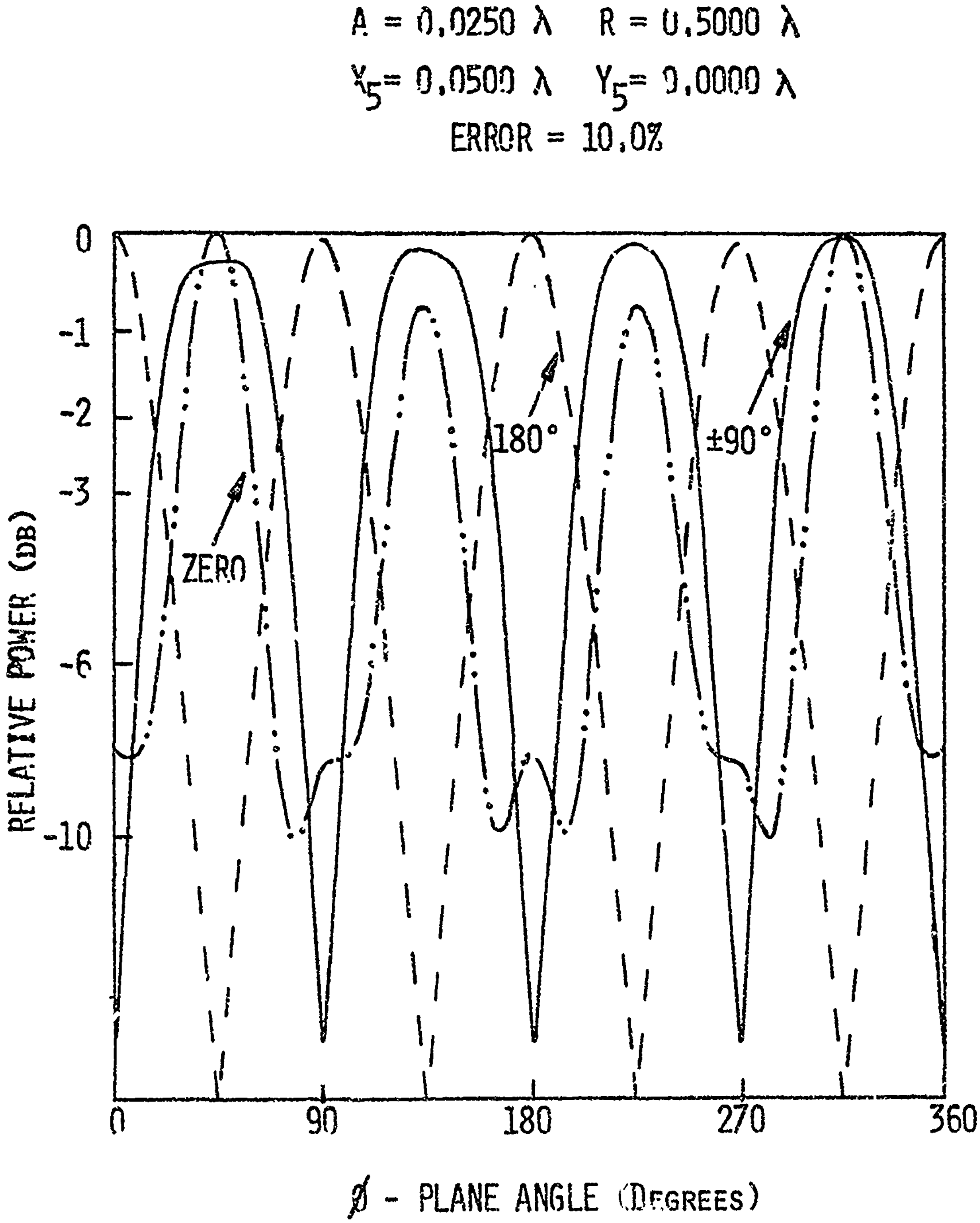


Figure 16 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

length. It is seen that lobes are formed at all phase progressions. As error increases, patterns are distorted in shape. The array with 0.5 wavelength radius is thus seen unsuitable for omnidirectional communications.

Figures 17 and 18 show patterns of the 0.3 wavelength radius array as in Figures 11 through 13 but the displacement of the center element is in the direction between two adjacent array elements (Ø = 45°) rather than towards one element (Ø = 0°). They are similar to those shown in Figures 11 through 13.

Other patterns including those with a different dipole radius are included in Appendix D.

5.5 Isolation

Isolation is defined as the ratio of power transferred to the 100 ohm load to the total power entering the array.

Power transferred to the 100 ohm load is equal to the product of the 100 ohm and the square of the feed point current of the center element. Power entering the array is the sum of powers entering each array element. Each array element is assumed to have a unit voltage across its feed point. Computer program for isolation can be found in Appendix A.

Results are shown in Figures 19 and 20. Figure 19 shows isolation as a function of position error. It is seen that except for the 0° progression, all other three progressions show increasing isolation with decreasing error. It approaches infinity asymptotically. The result verifies the trend of input impedance discussed in section 5.3.

Figure 20 shows the effect of dipole radius on isolation. For the range of radii investigated, isolation is insensitive to dipole radius.

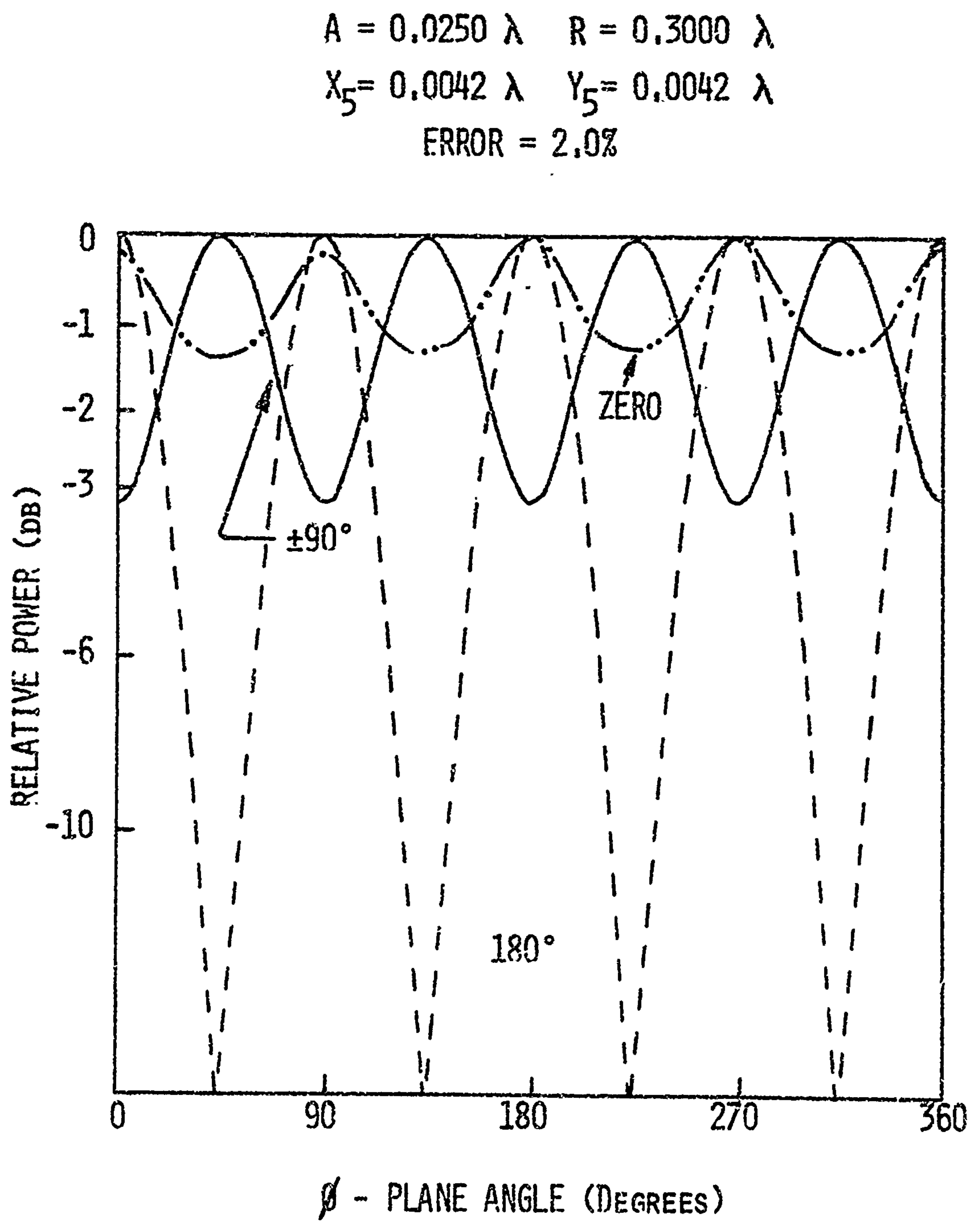


Figure 17 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

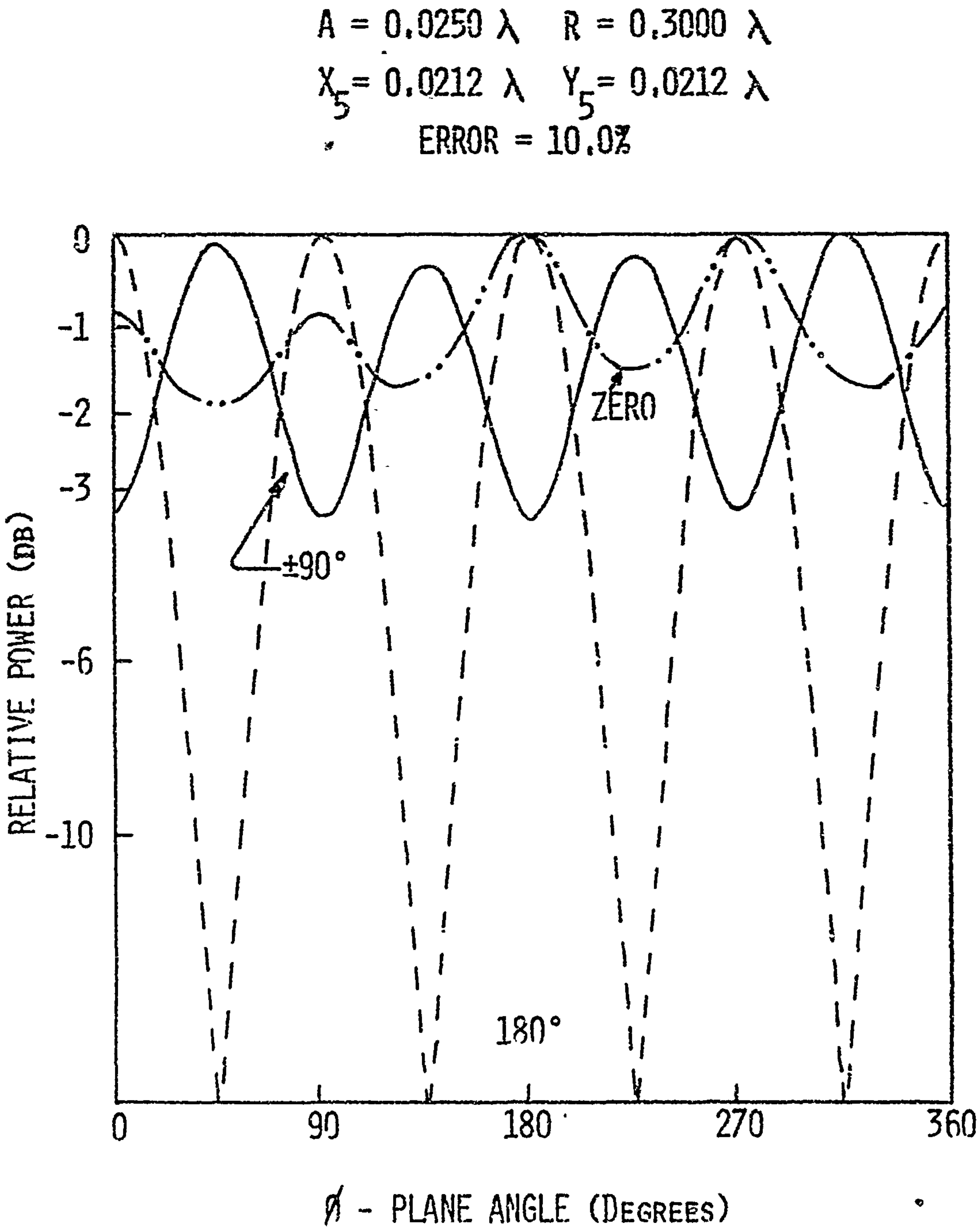


Figure 18 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

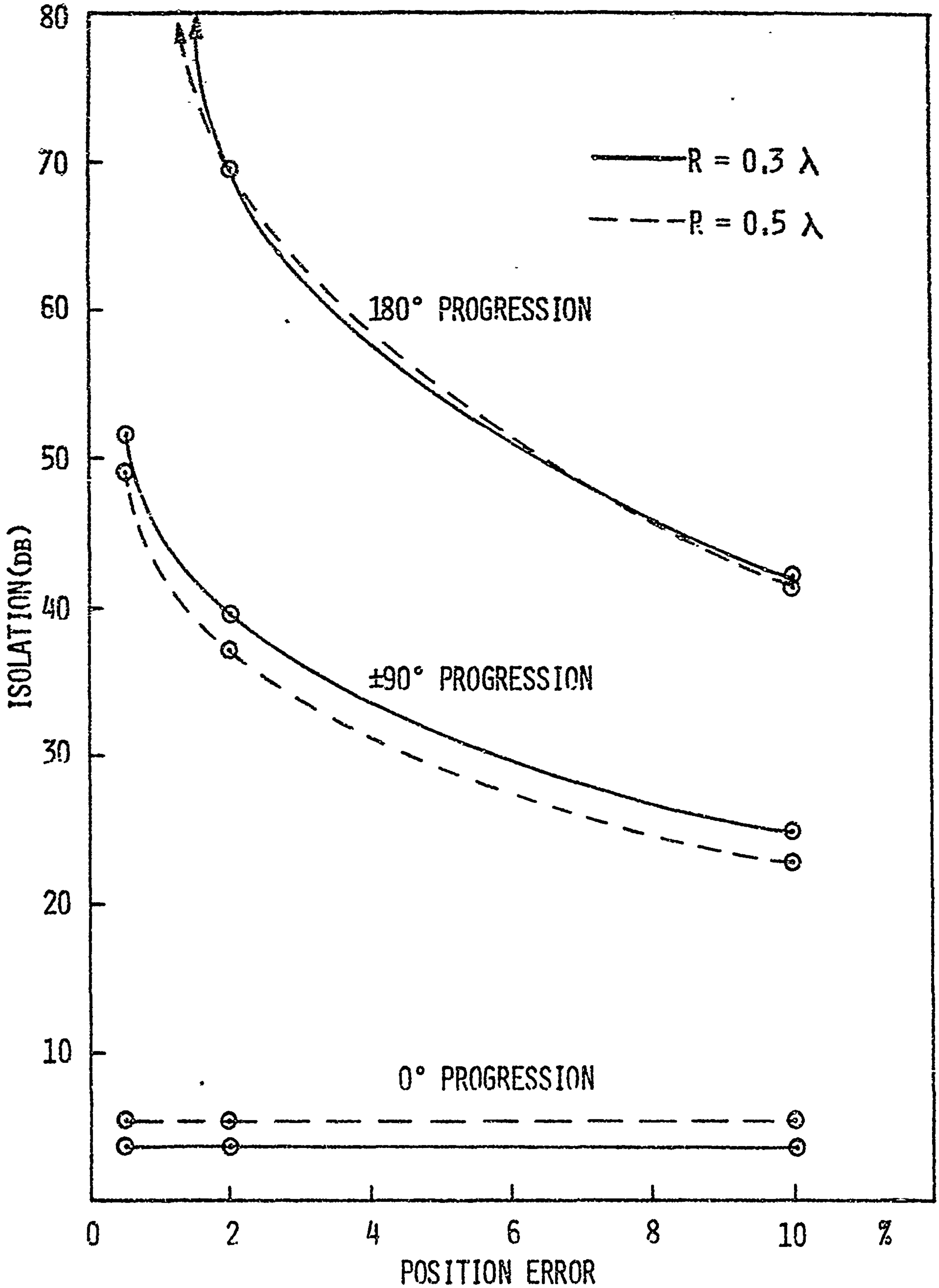


Figure 19 : Isolation as a Function of Center Element's Position Error for Various Phase Progression and Array Radii.

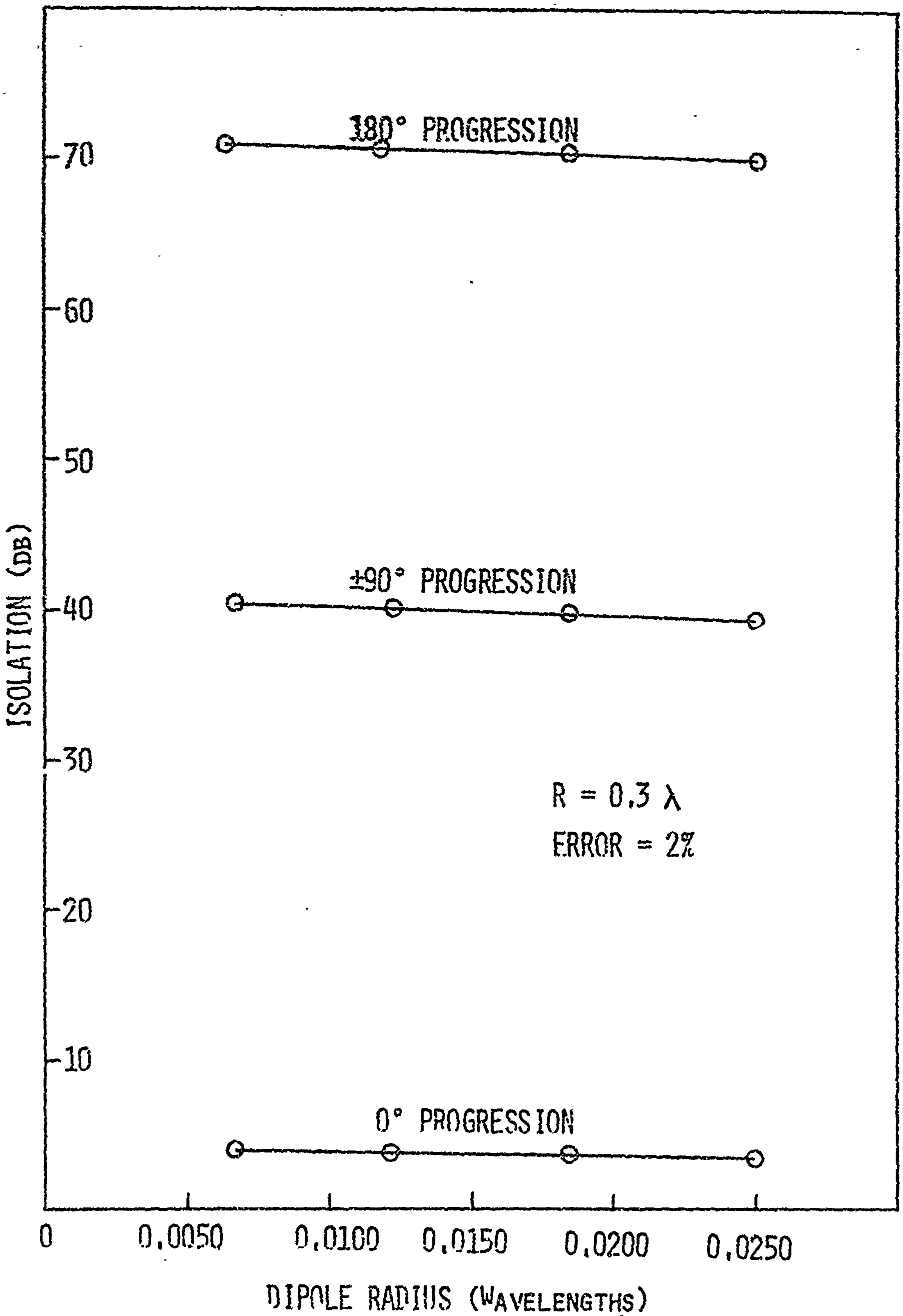


Figure 20 : Isolation as a Function of Dipole Radius.

6. EXPERIMENTAL STUDY

A laboratory setup was assembled using scaled models of existing FAA antennas and a power dividing and phasing network. This section is divided to cover antenna and network design, radiation characteristics, isolation, gain, and other capabilities of the array.

6.1 Antenna and Network Design

An FAA coaxial dipole was used as the basis of design. The FAA dipole was originally designed for use at VHF. By keeping the proportion, the dipole was scaled down in size to L-band, with a center frequency of 1.45 GHz. The mechanical drawing of the scaled antenna is shown in Figure 21. An exploded view of the scaled dipole is shown in Figure 22. The assembled dipoles are shown in Figure 23.

VSWR of dipoles were measured. VSWR plot of the original VHF dipole is shown in Figure 24, and the plot of scale models is shown in Figure 25. It is seen that general shapes of the plots are similar, with the exception that the VHF dipole has a much narrower bandwidth but a better match at center frequency. This is because the VHF antenna tested had a matching network at the feed point. The matching network of the original dipole consists of a single coil. Because of that matching network, the antenna is tuned to a single frequency and exhibits a narrower bandwidth characteristic there. Since a perfect match is not required to prove the theory of the circular array, the matching network was not used for the scale model.

For ease of controlling phase distribution, a simple power divider system incorporated with variable phase shifters was used. The circuit diagram is shown in Figure 26. The input power is fed in from the top and is first divided into two equal parts by an isolated power divider then further divided into four parts by two more isolated power dividers. Attached to each of the four arms of the divider is a trombone line stretcher. After the line stretcher, the line feeds into an antenna. Figure 27 shows the picture of such a network.

The accuracy of phase adjustment was within ±5°

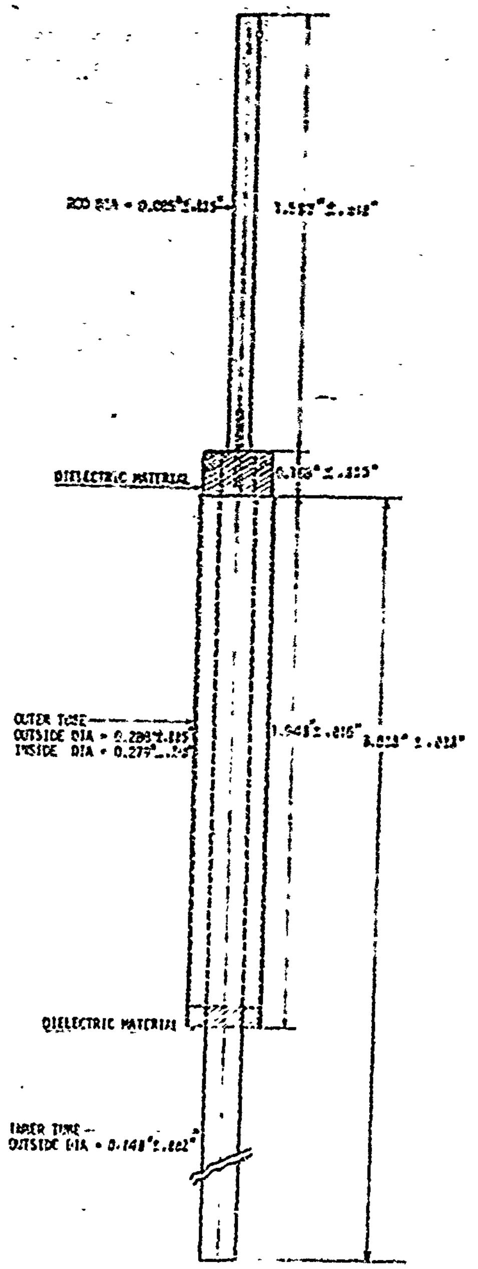


Figure 21 : The Mechanical Drawing of The Scaled Coaxial Dipole.

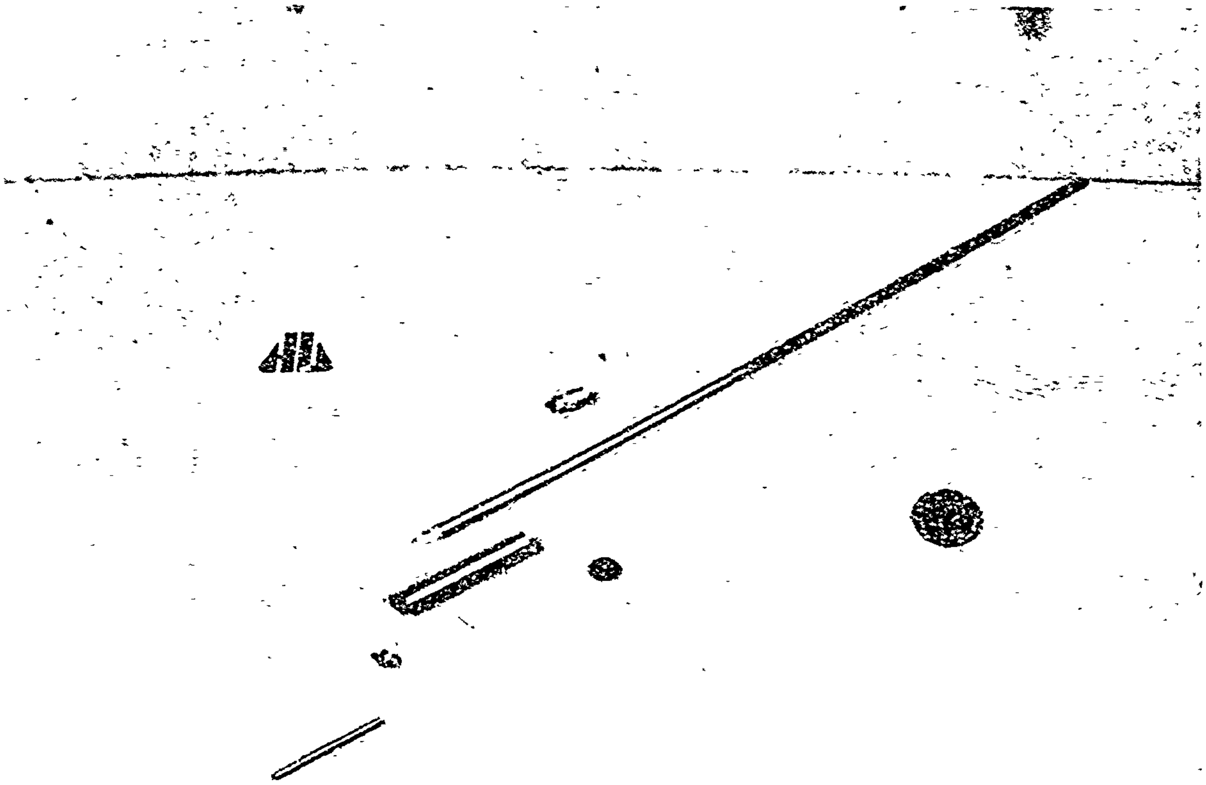


Figure 22 : An Exploded View of the Scaled Dipole.

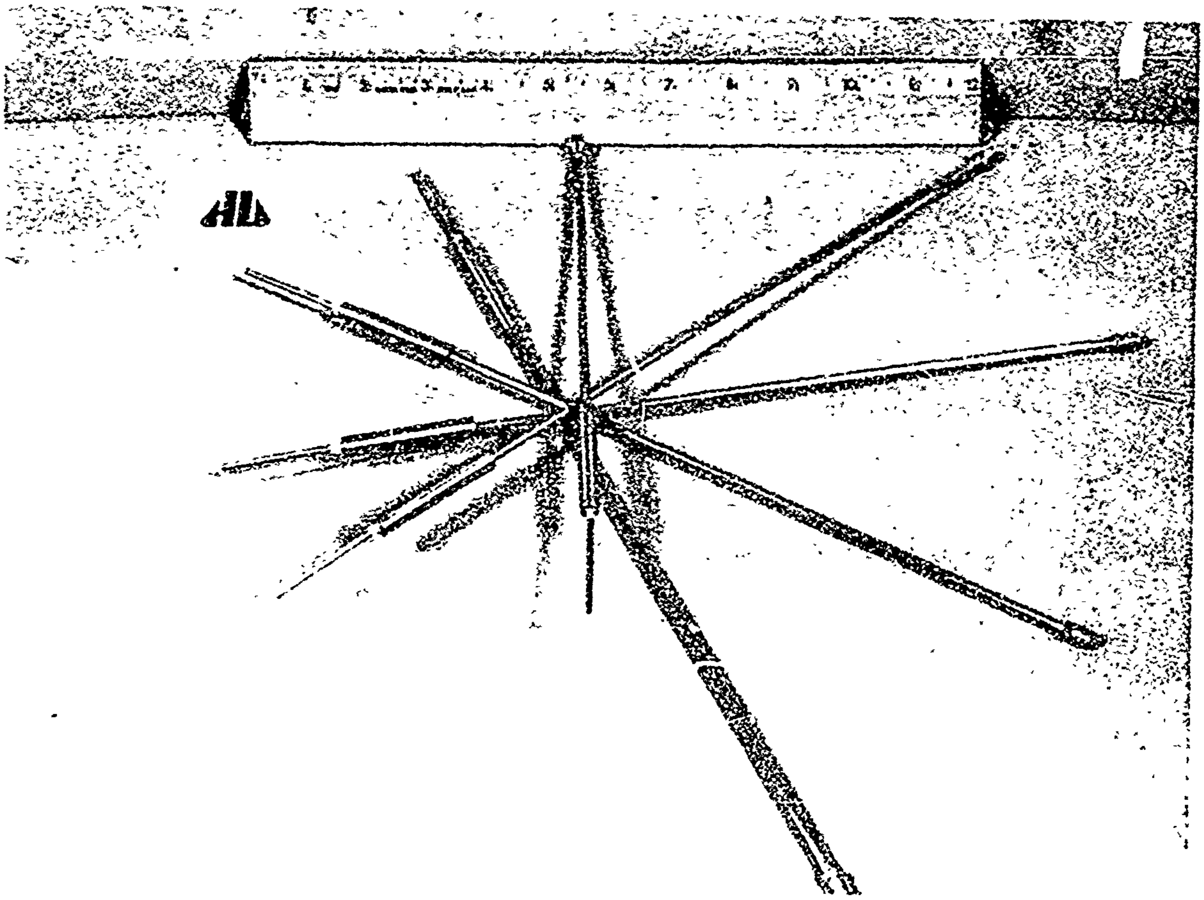


Figure 23 : Assembled Scaled Dipoles.

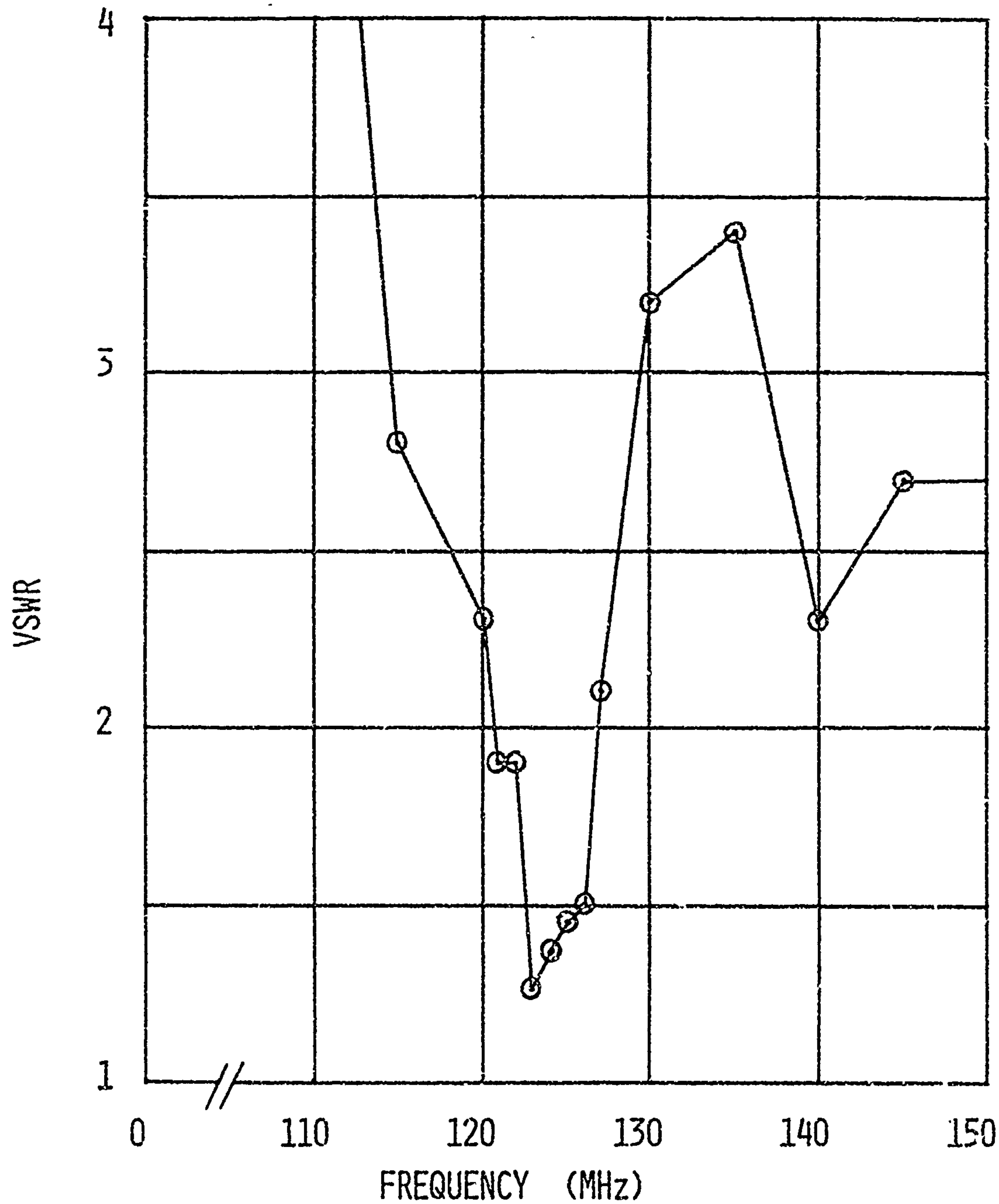


Figure 24: VSWR of VHF Dipole Type FA-5248.

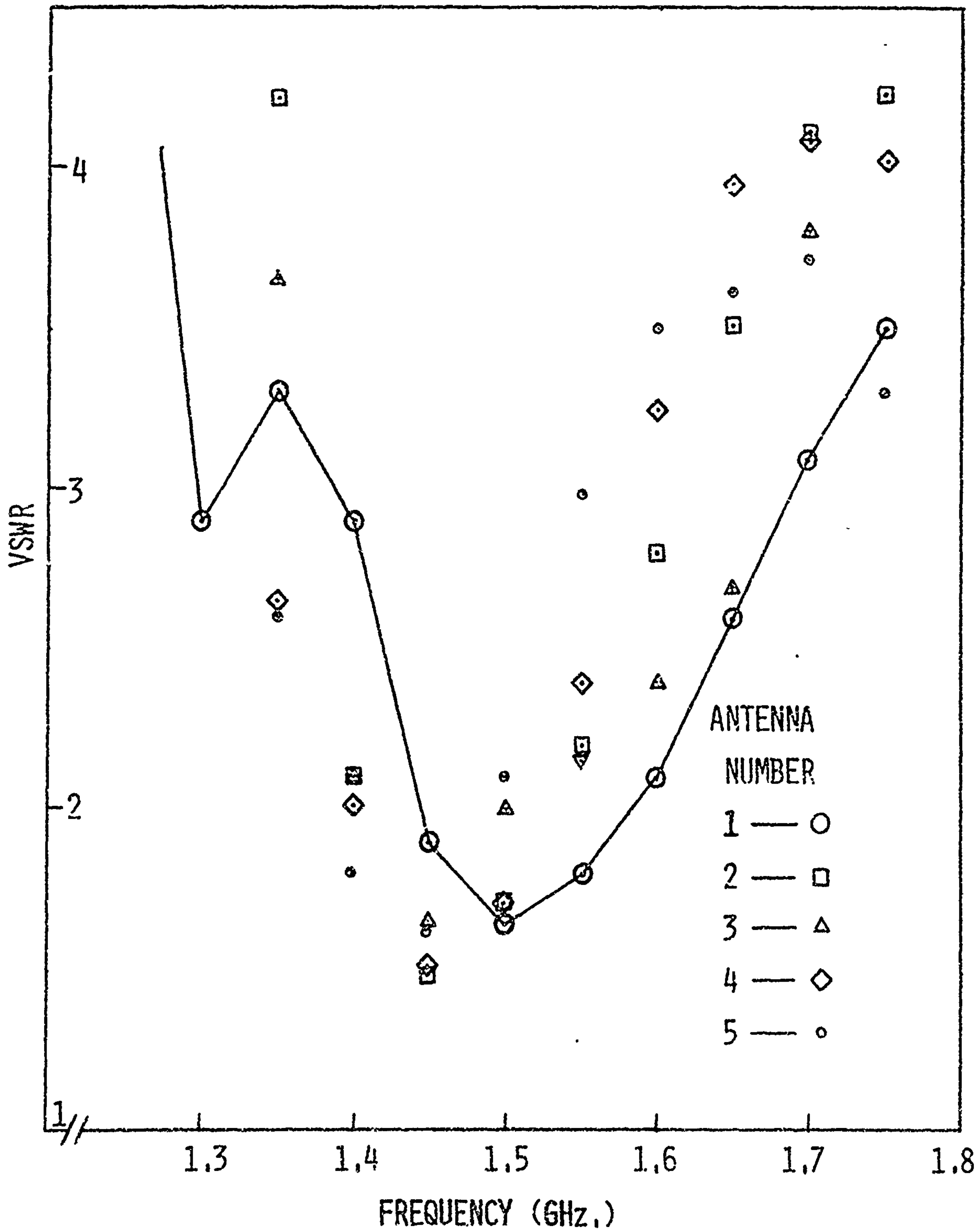


Figure 25 : Measured VSWR of Scale Dipoles as a Function of Frequency.

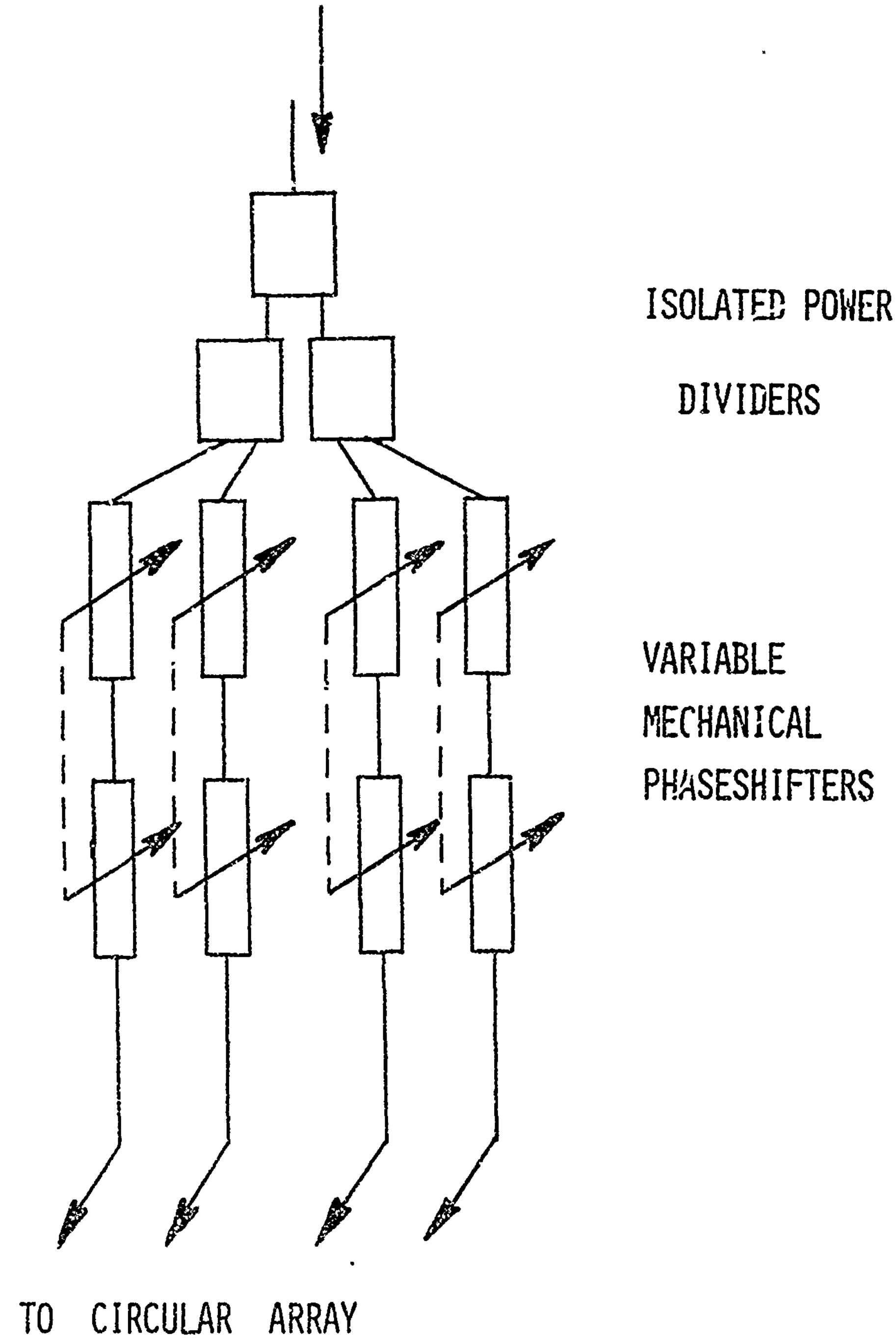


Figure 26: Circuit Diagram of the Feed Network.

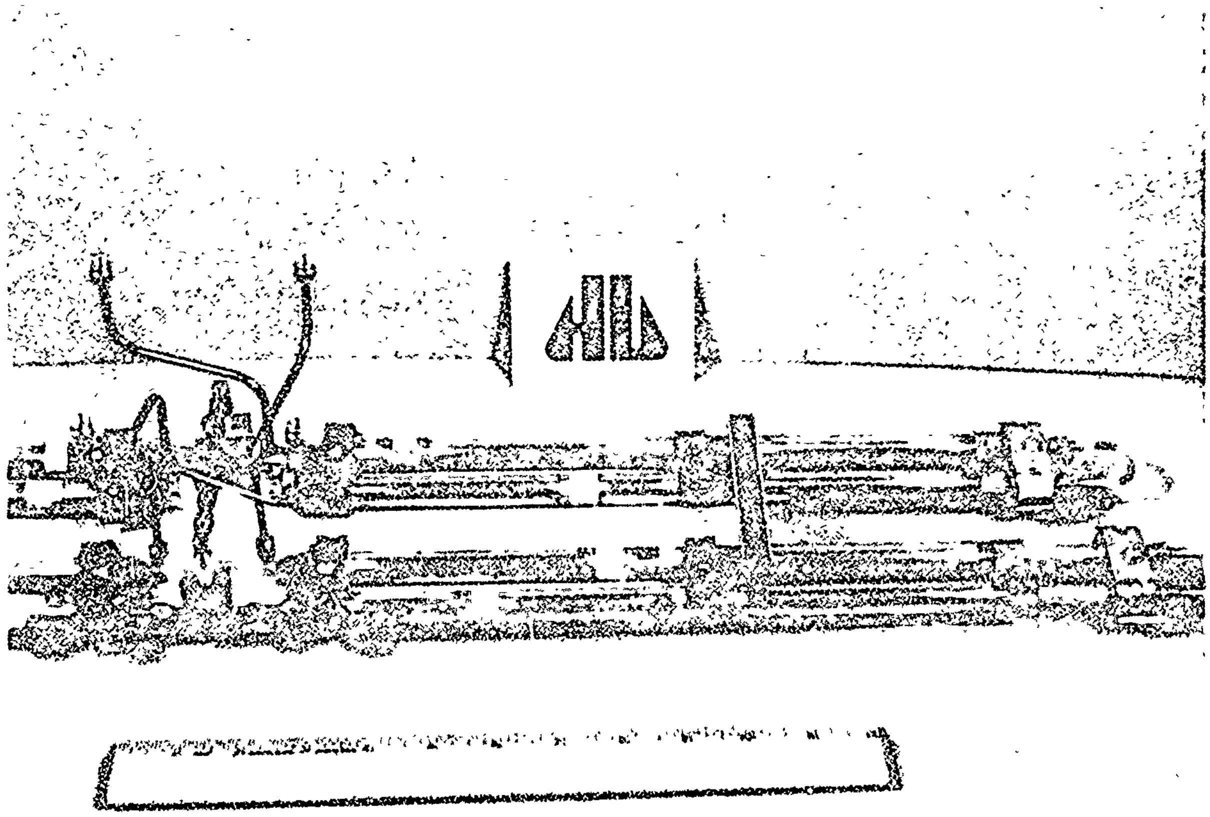


Figure 27: The Feed Network

and the accuracy of power distribution was within ±1 db.

A circular array with a diameter of 0.6 wavelength was formed using above described elements and feeding network. The array itself was supported by rigid dielectric braces. The trombone phase shifters were adjusted to give a 90° phase progression. The antenna feed arrangement is depicted in Figure 28, where element No.1 is fed with unit amplitude and 0° phase, element No.2 is fed with unit amplitude and 90° phase, element No.3 is fed with unit amplitude and 180° phase, and the last one is fed with 270° phase. The element located at the center is the test element which is supposed to be isolated from the circular array by virtue of the array's phase progression. The system was then tested, and results discussed in following sections.

6.2 Radiation Characteristics

The array was tested for three different phase progressions. Array patterns and center element patterns were plotted as well as other variations. They are discussed below.

6.2.1 The 90° progression

Radiation patterns of the 90° progression are shown in Figure 29. The array pattern has a peak to null variation of ±3.5 db. But part of that variation was caused by network amplitude unbalance, which shows up as a wave in the pattern. If the network is balanced and the wave is taken out from the pattern, it will leave only the ripples, and the variation will be reduced to ±2 db.

Because of the array blockage, the pattern of the center element is seen to have ripples too. The pattern is shown in the same figure. But it is seen that the pattern has less variation than the array pattern.

When the center element position error is increased from 2% to 10%, it is found that the array pattern stays practically the same, but the pattern of the center element has changed in two ways. First of all the relative amplitude has decreased in some

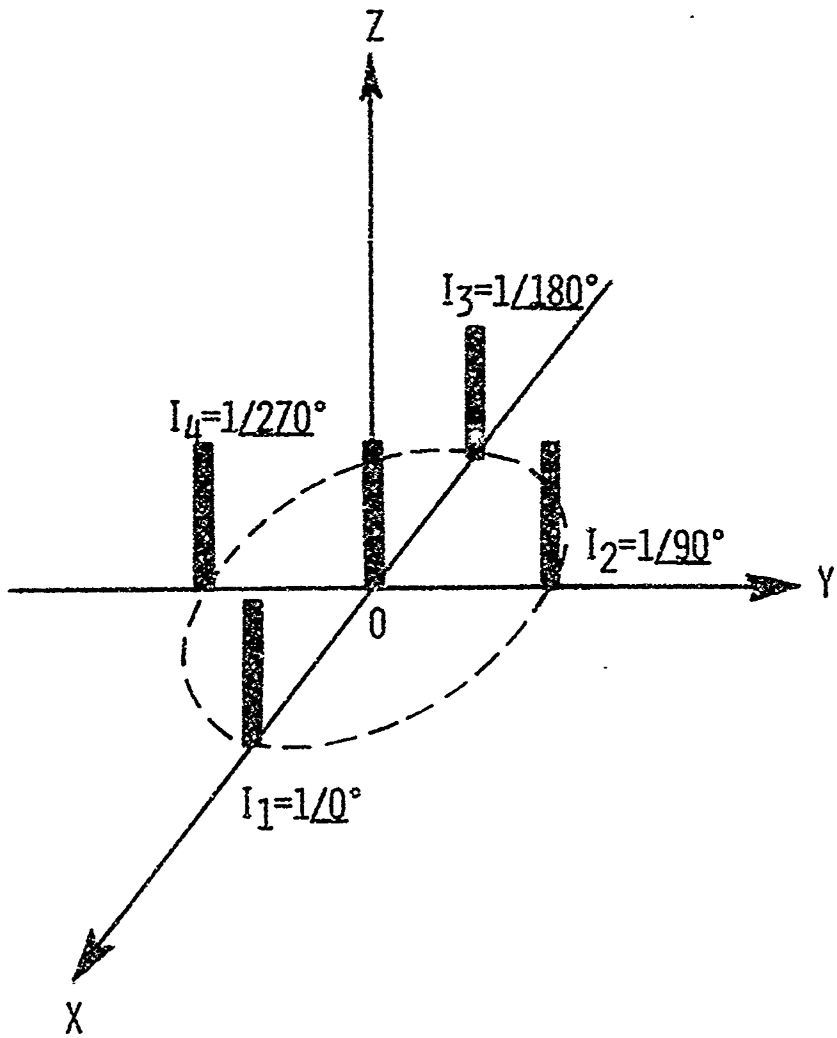


Figure 28 : The Array Feed Sequence Illustrating
The 90° Progression.

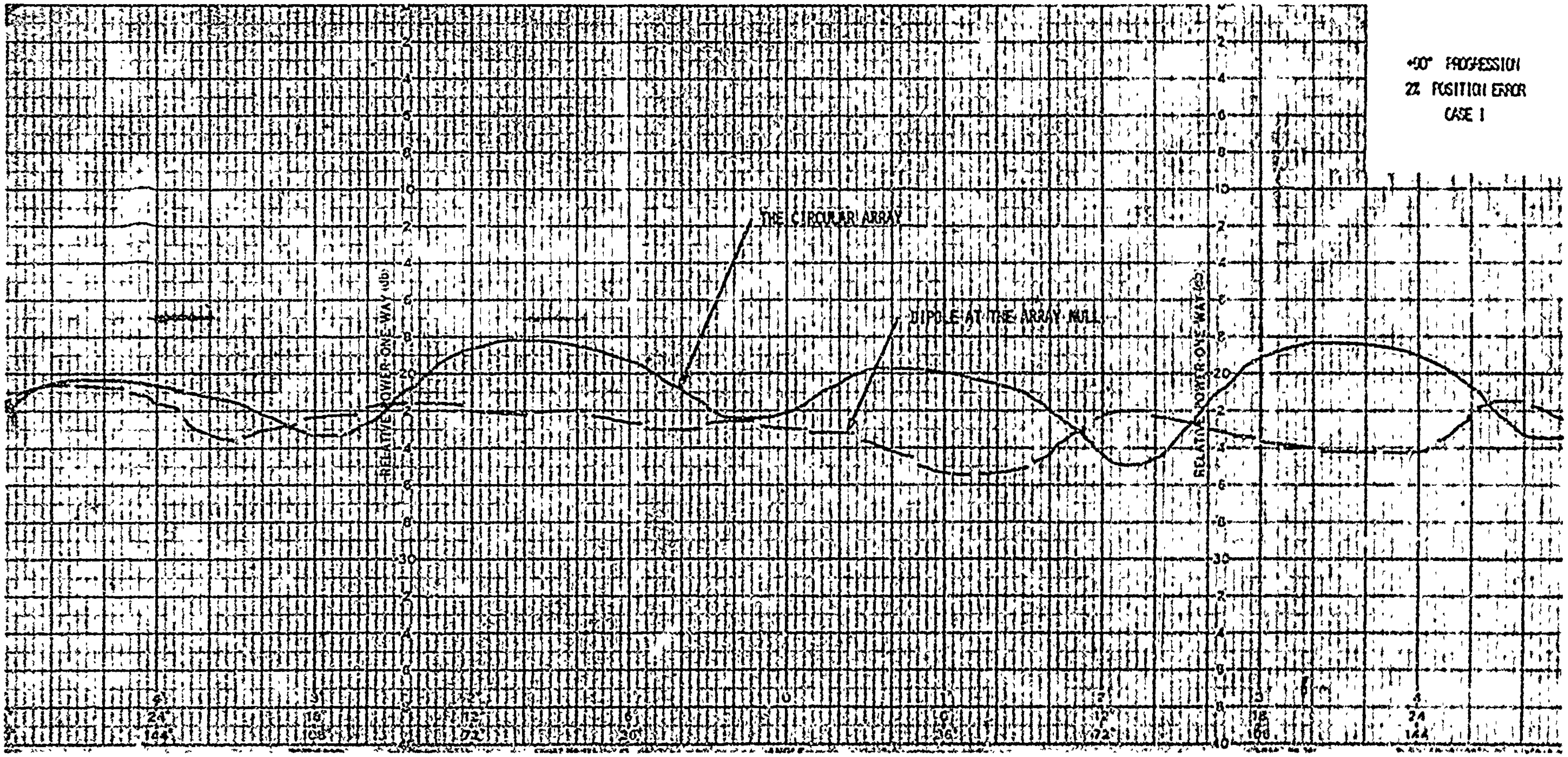


Figure 29 : Radiation Patterns of a 90° Phase Progression circular Array, and an Isolated Element in the Array Center. The Array Radius is 0.3 λ. The Isolated Element is 2% off the Null Point.

spots, and then the pattern is less isotropic. The patterns are shown in Figure 30. The reason for the changes is because the array is now coupled to the center element, closer than before, due to an increased position error of the center element.

In a reverse manner, the pattern of the center element will be more isotropic and has a higher gain if coupling is less. This will be shown to be true in section 6.2.3.

6.2.2 Other Progressions

The feed network was adjusted for 0° and 180° phase progressions. Figure 31 shows the result of 0° progression. Because of strong coupling, both the array and center element patterns deviate from being isotropic. Figure 32 shows the result of 180° progression. The array pattern shows four nulls, as depicted by the theory.

6.2.3 Elevated Center Element

In order to observe the effect of higher isolation, the center element was elevated from the array plane by a quarter wavelength. The isolation of the 90° progression was found to have improved by 3 db, and radiation patterns were plotted in Figure 33. An improvement is seen in the center element pattern as expected and as explained in section 6.2.1.

6.2.4 Effect of Other Obstacles

In order to study the effect of obstacles, loaded and short-circuited antennas, and a piece of straight wire were used in succession to simulate various situations. Figures 34 through 36 show the radiation patterns of a single coaxial dipole in the neighborhood of a loaded dipole, a piece of straight wire, and a short-circuited dipole respectively. Pattern distortion is seen to become worse. As a comparison, Figure 37 shows the pattern of a 90° progression array one wavelength away from a loaded dipole, measured from the array edge. Distortion is negligible. The reason is partly because the spacing is larger, but the other reason could be because the array itself is 0.6 wavelength in diameter, thus the effect of the obstacle tends to be distributed

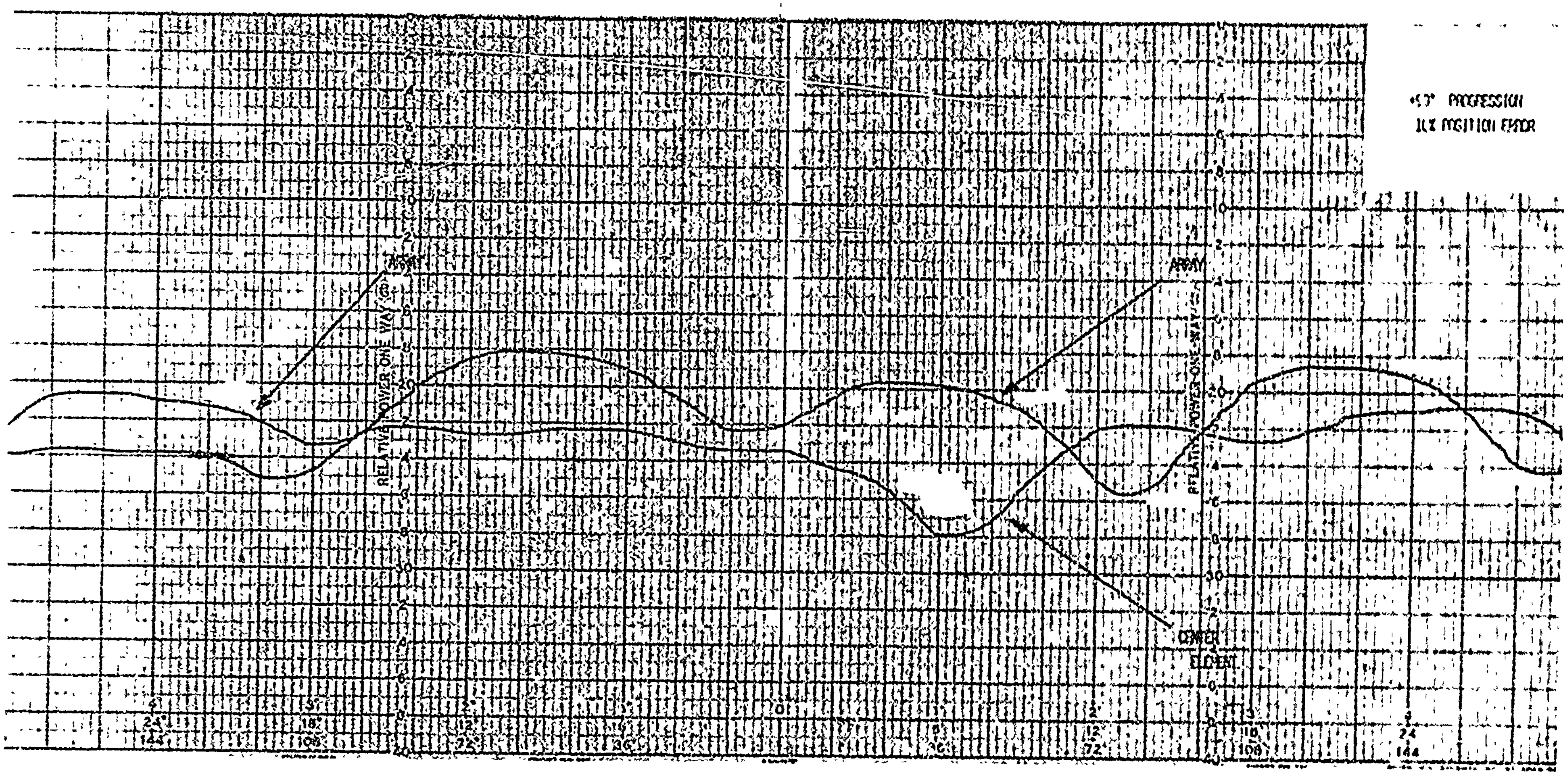


Figure 30 : Radiation Patterns of a 90° Phase Progression circular Array, and an Isolated Element in the Array Center. The Array Radius is 0.3 λ. The Isolated Element is 10% off the Null Point.

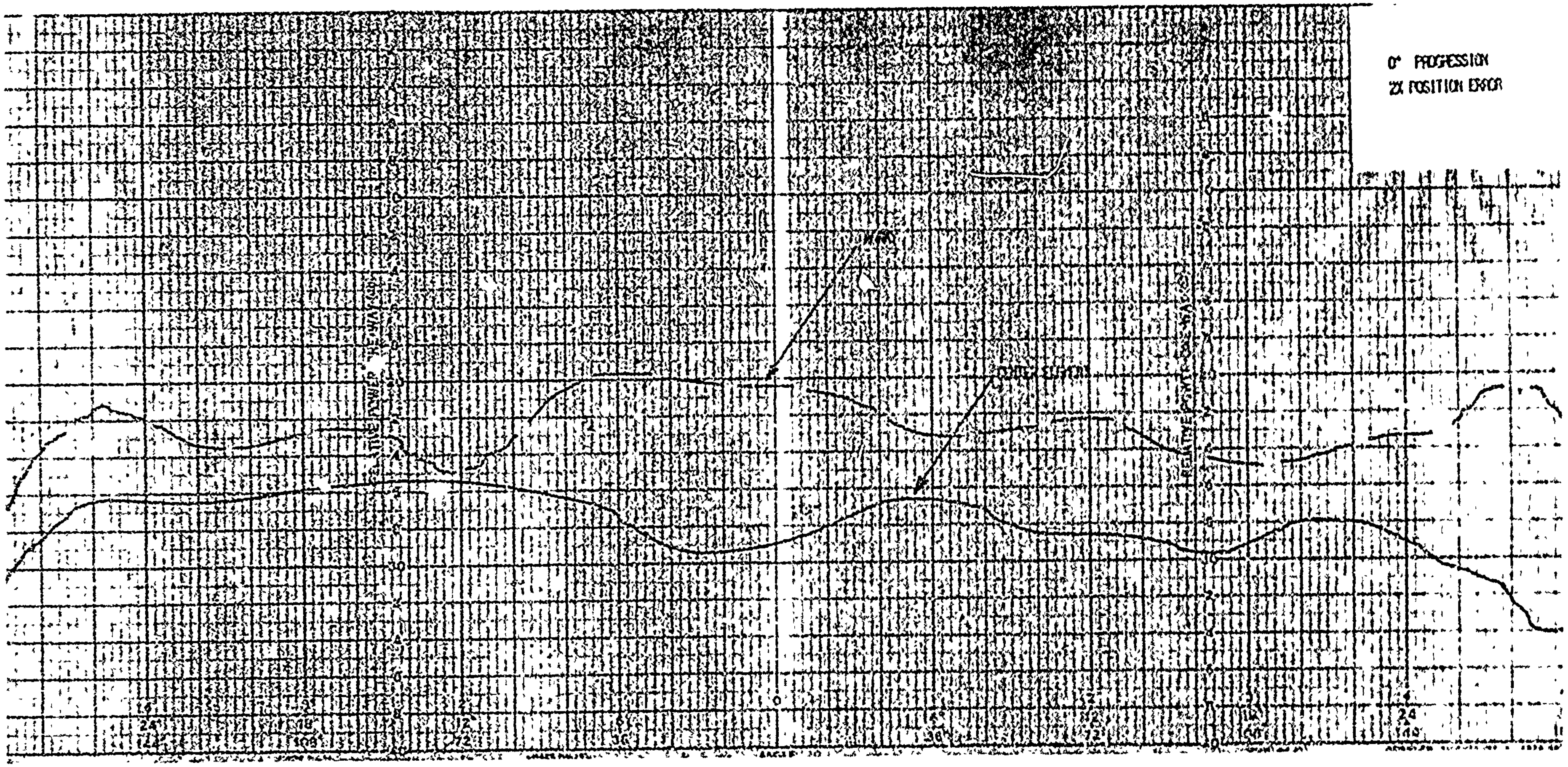


Figure 31 : Radiation Patterns of a 0° Phase Progression Circular Array, and an Isolated Element in the Array Center. The Array Radius is 0.3 λ. The Isolated Element is 2% off Center.

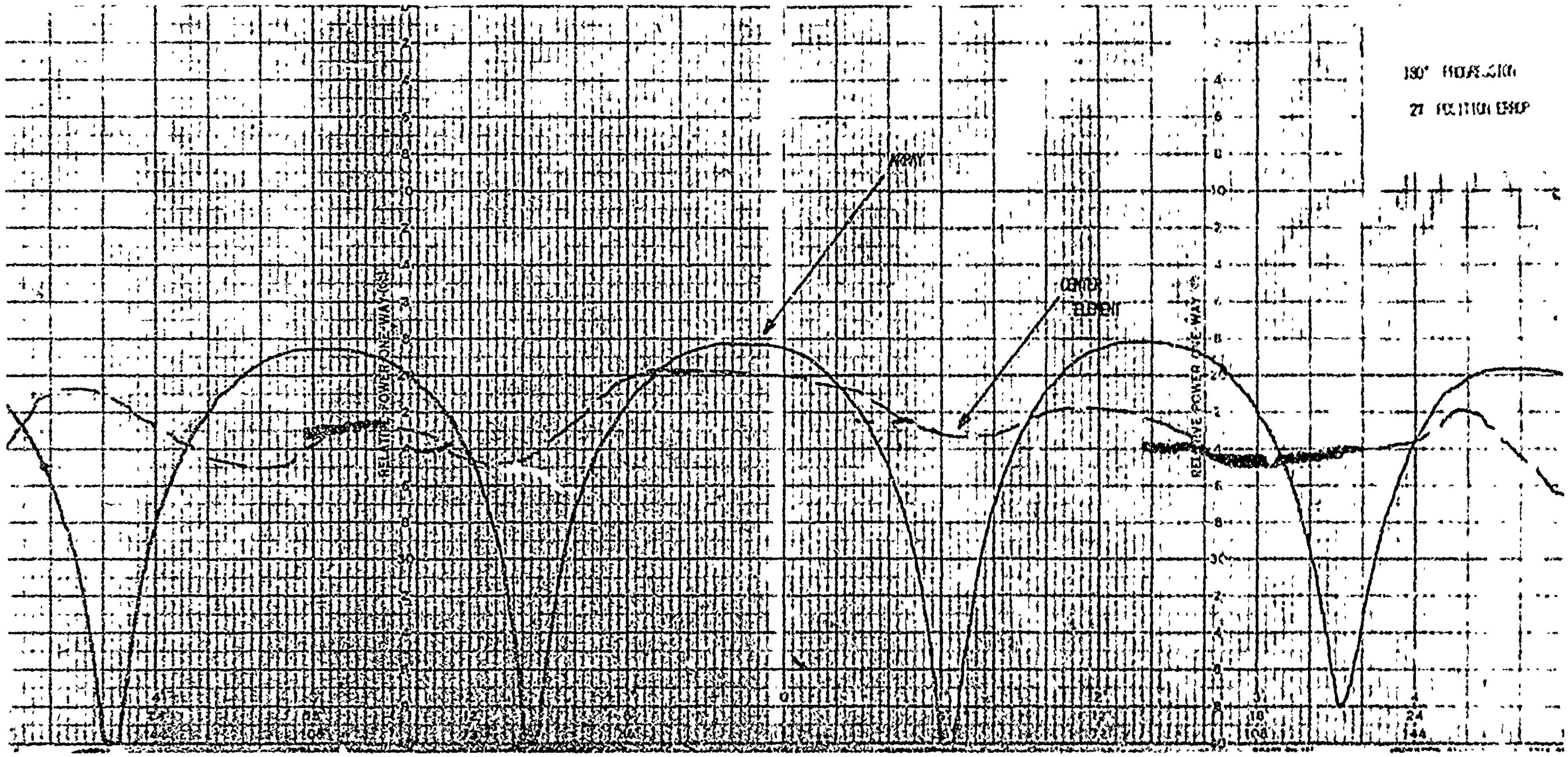


Figure 32 : Radiation Patterns of a 180° Phase Progression circular Array, and an Isolated Element in the Array Center. The Array Radius is 0.3 λ. The Isolated Element is 2% off the Null Point.

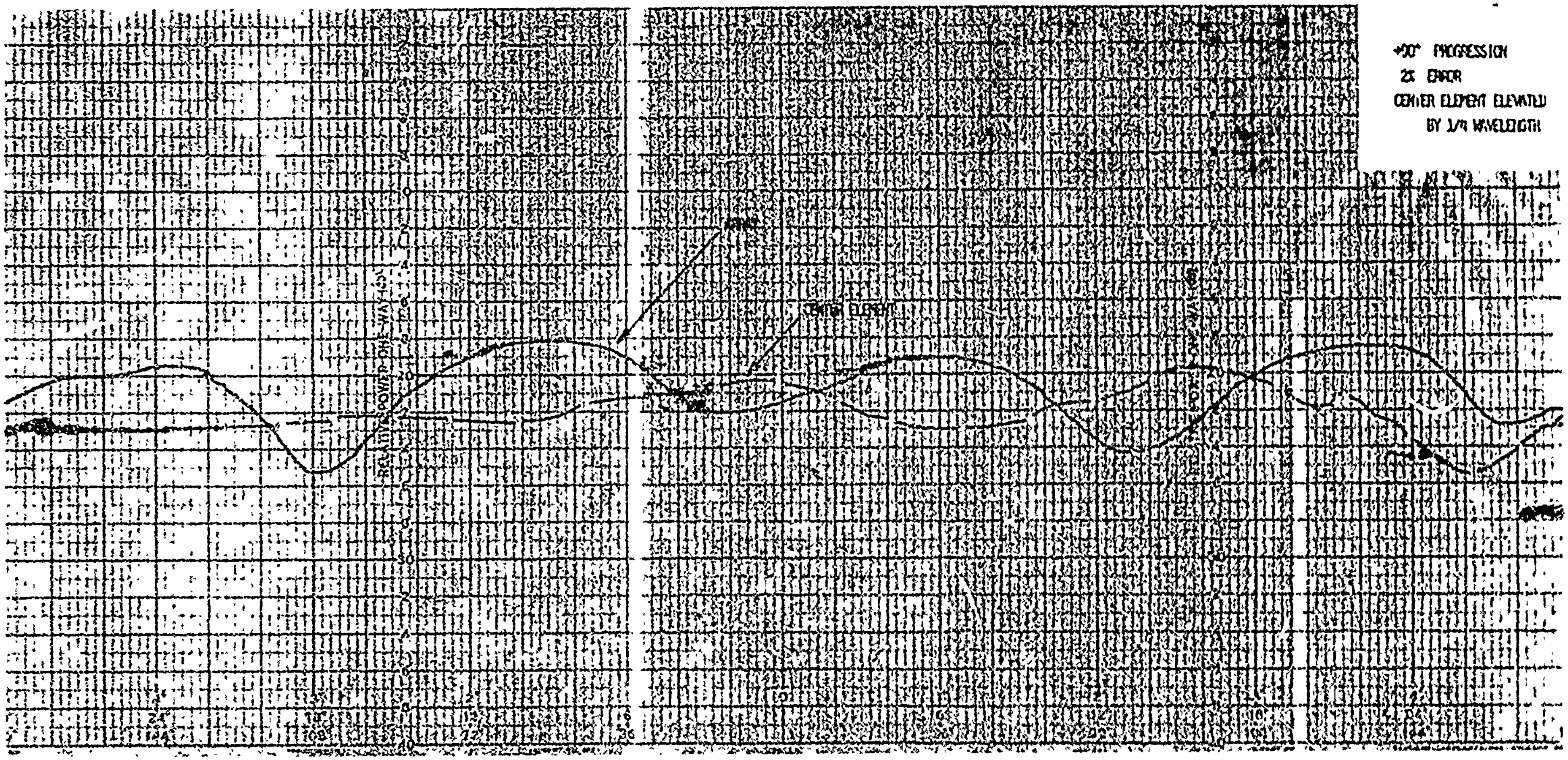


Figure 33 : Array and Center Element Patterns When the Center Element is Elevated by $\lambda/4$.

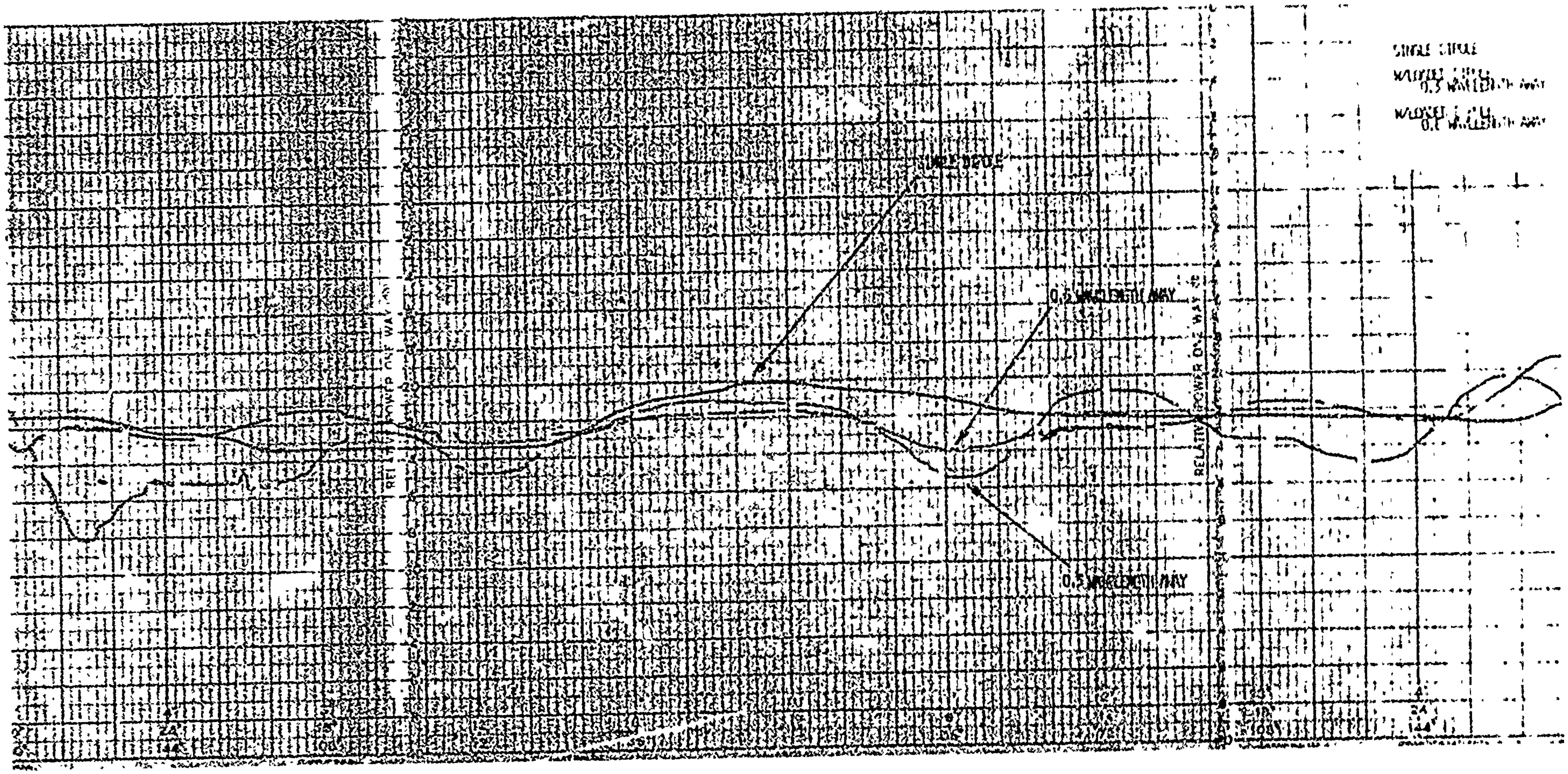


Figure 34 : Radiation Patterns of a Single Sealed Dipole with a Loaded Dipole at various Distances Away.

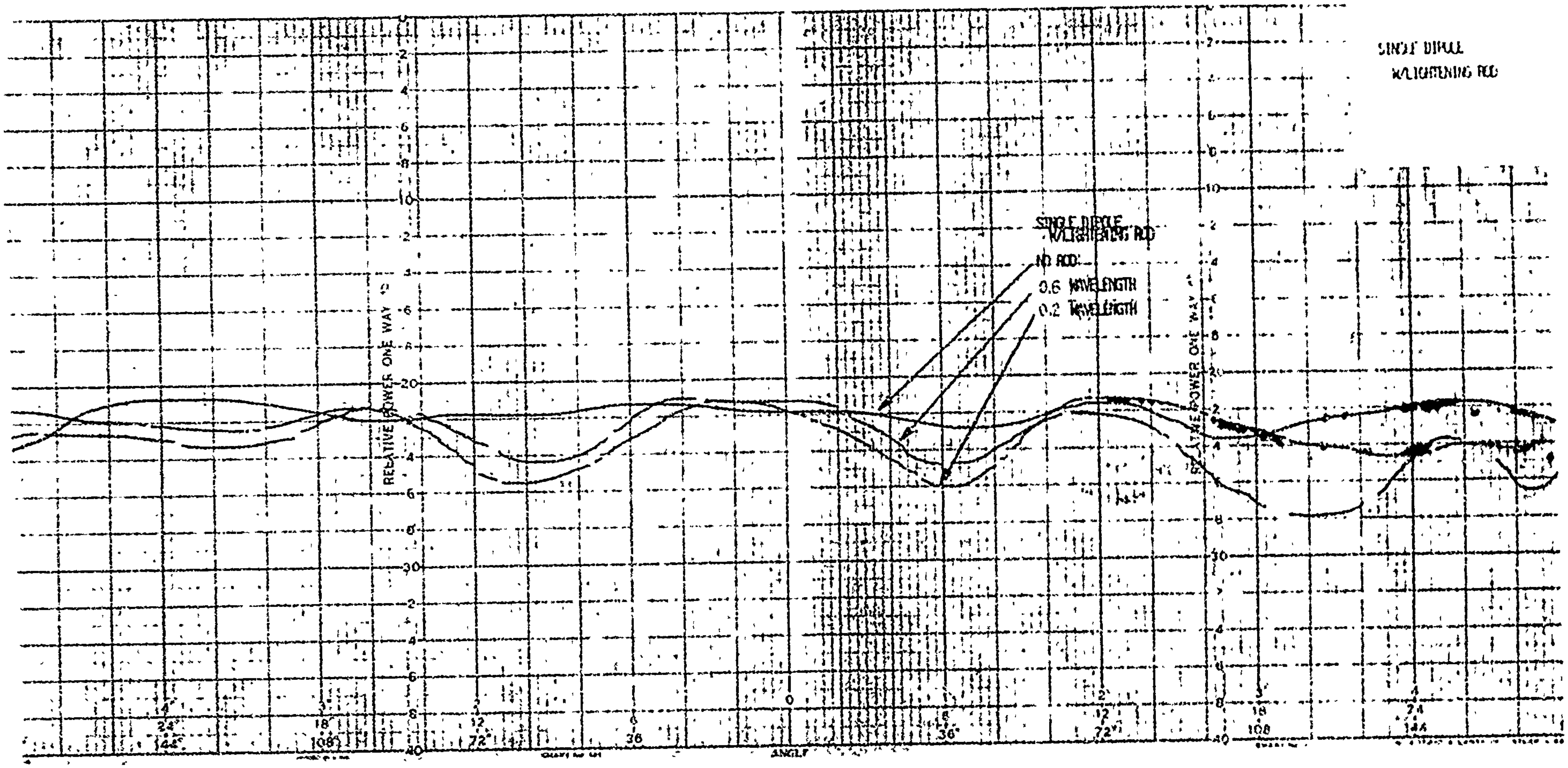


Figure 35 : Radiation Patterns of a Single Dipole with a Lighting Rod at various Distances Away.

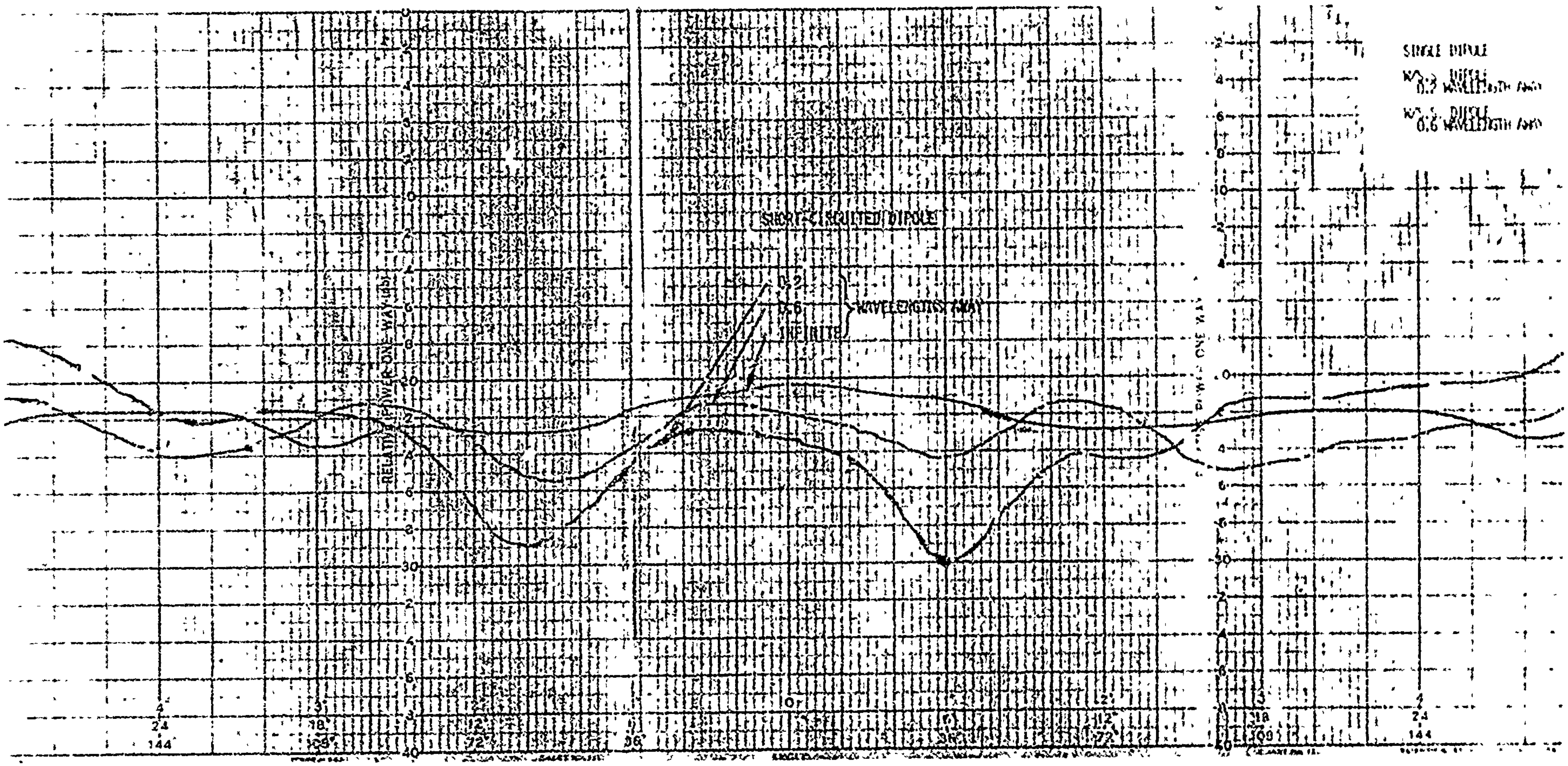


Figure 36 : Radiation Patterns of a Single Dipole with a Short-Circular Dipole at various Distances Away.

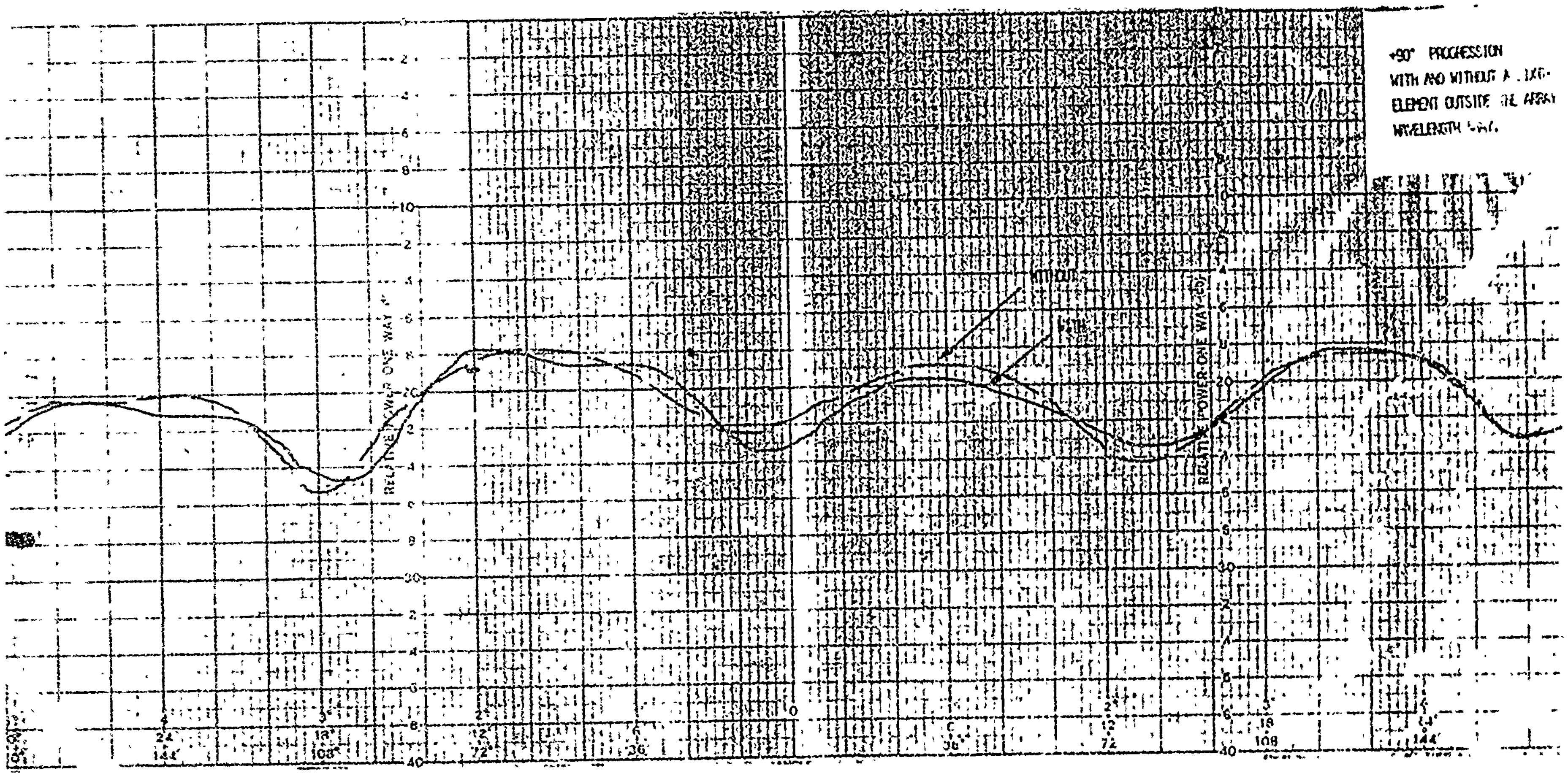


Figure 37 : Radiation Patterns of a 90° Progression Array with and without a Sixth Element Positioned One Wavelength Away.

among array elements, rather than concentrated in one element.

6.3 Isolation

For the 90° phase progression, a series of isolation data points were taken and mapped onto the array plane. Using linear extrapolation between points, an equal-isolation contour plot is generated. Using the extrapolated null point as the origin and the lines connecting it to antenna locations as the axis, displacement away from the null was marked off along the axis. Isolation values read from this plot compares closely with theoretical results, especially the trend of increasing isolation as the position error is decreased. The plot is shown in Figure 38.

The difference between theory and experiment is largely due to:

1. The theory assumes symmetrical dipoles, whereas the experiment used coaxial dipoles.
2. Network loss is present.
3. The effect of an absorber ground plane which is placed one wavelength away from the bottom end of the dipole to minimize the effect of supporting structures may be present.

6.3.1 Value of High Isolation

A high isolation of 55 db was measured for the 90° progression. Higher values can be achieved if mechanical tolerances can be held.

There are two types of isolations that exist. One is the isolation between the array and the center element. The other is the isolation between one array progression and the other. The first is controlled primarily by position precision. Other factors that do enter into the consideration are array phase precision and array amplitude precision.

Isolation between progressions is determined mainly by the feed network matrix's isolation. At present the best estimate of achievable matrix isolation is about 30 db for a transmission line network and 40 db for a waveguide network.

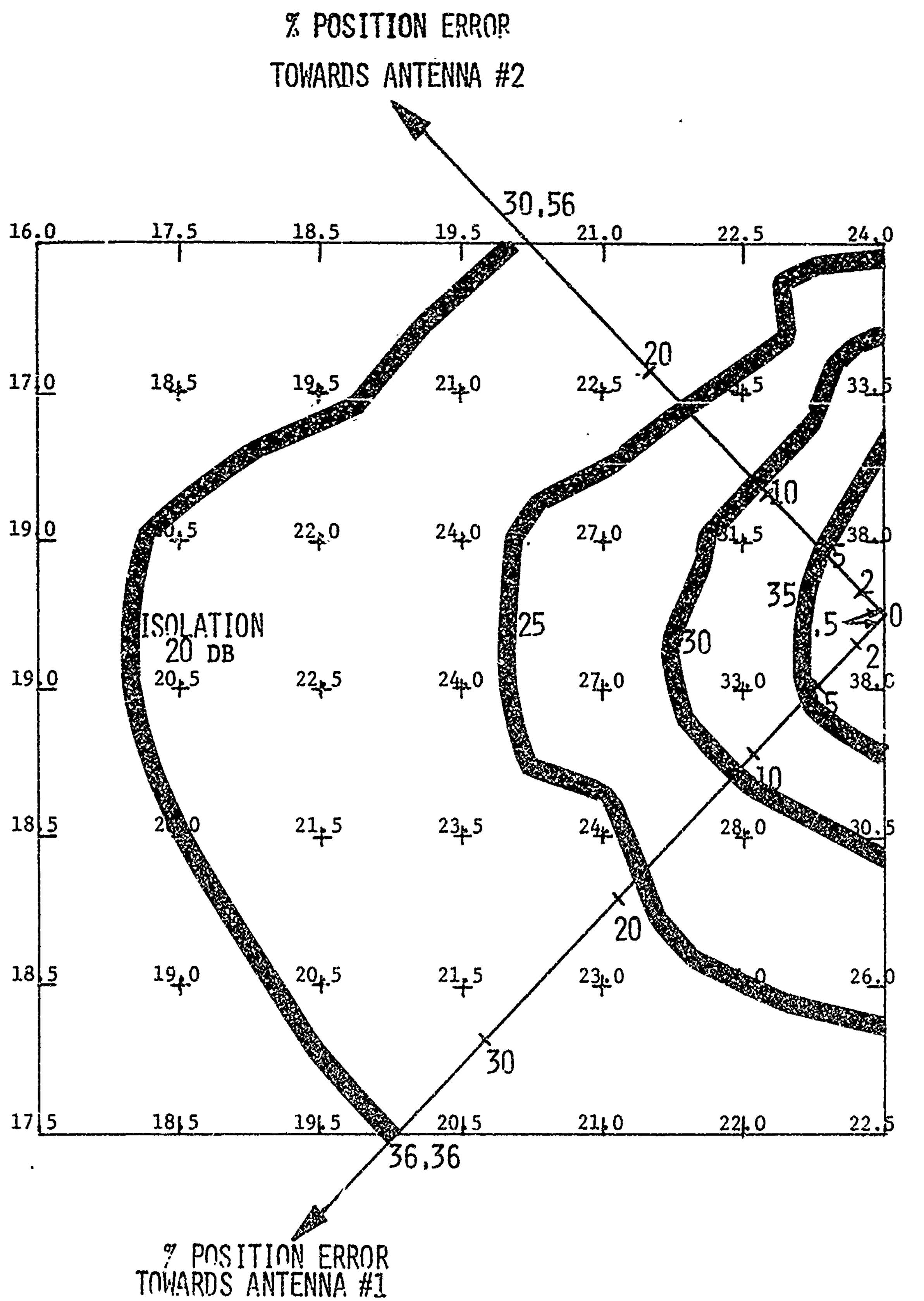


Figure 38: Equal-Isolation Contour Plot for 90° Phase Progression.

6.3.2 Isolation Between Two Half-Wave Dipoles

In existing FAA facilities, the model of two dipoles can adequately depict the isolation between antennas. Figure 39 shows the theoretical plot of dipole isolation as a function of spacing. It is seen that in order to get 30 db isolation, the dipoles have to be spaced 4 wavelengths apart, and to get 55 db isolation, the two dipoles cannot be mounted on the same FAA tower or similar structures of that size. In contrast to that, the circular array needs no more room than a circular aperture 0.6 wavelengths in diameter.

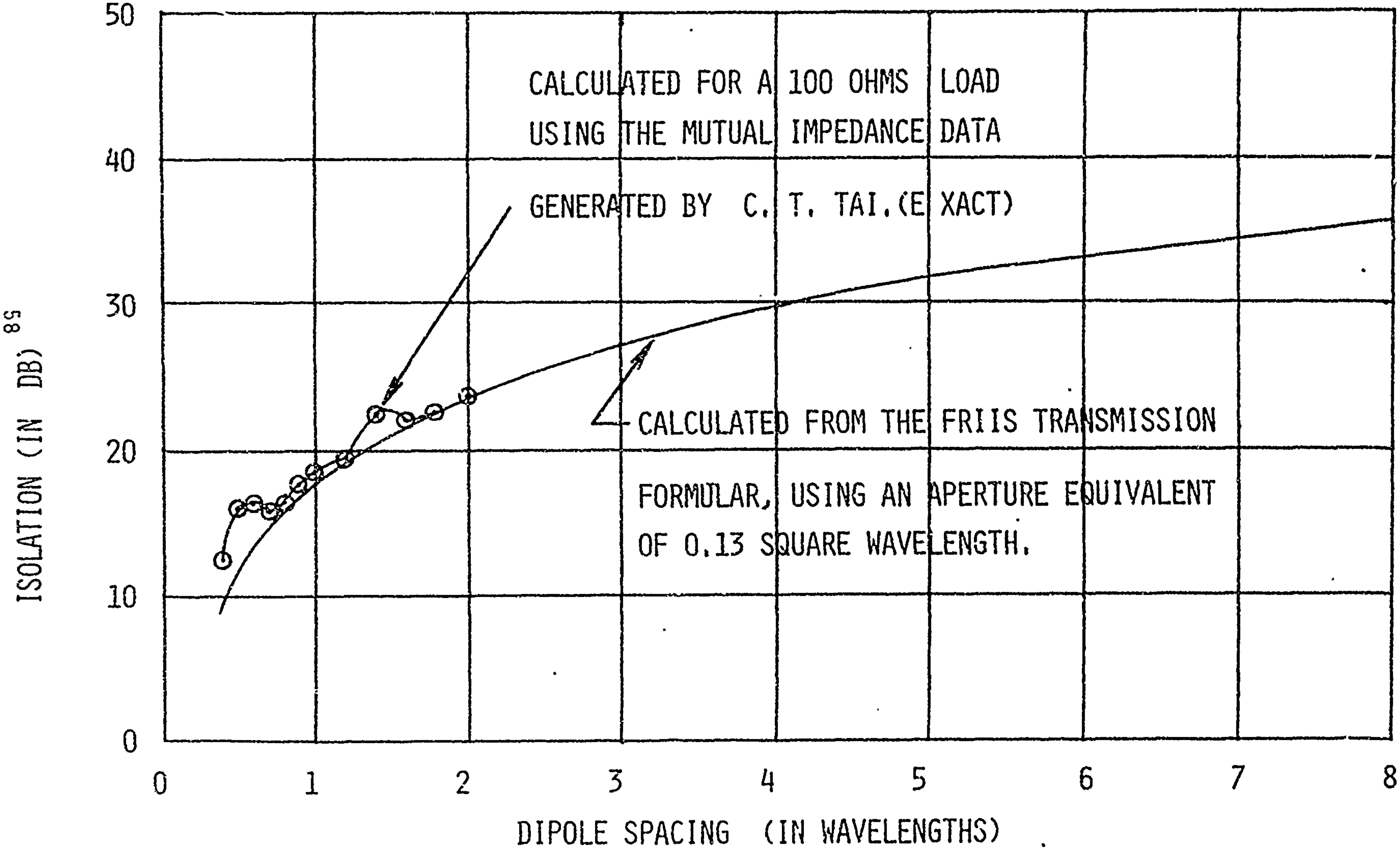
6.4 Gain

The circular array gain at 90° progression was measured. The measurement made use of the insertion loss technique and Friis transmission formular. The gain was found to be 0.5 db above an omnidirectional antenna. This gain value includes the feed network loss, and the coupling loss to the center element.

6.5 Other Capabilities

Because the array is symmetrical about the vertical axis, uniform scanning can be expected if future needs require such capability. Beam forming, and simultaneous beam scanning are all possible.

Figure 39: Isolation Between Two Half-Wave Dipoles as a Function of Spacing.



7. CONCLUSIONS

It is essential to improve antenna isolation in order to up-grade present FAA communication systems. The concept of using progressively phased circular arrays to achieve high antenna isolation has been investigated theoretically and experimentally. It is found to be technically feasible. Isolation as high as 55 db was achieved. Pattern distortion is not severe, as compared to distortions caused by nearby obstacles in existing systems. Space needed to mount the system is one order of magnitude less than what is required at present for the same isolation.

Disadvantages as compared to using single dipoles are found to be that isolation depends on position accuracy. That problem can be solved by building the array as a module. If weather resistance is required, a radome can be used. Because of the feeding network, the array is lossier. The network loss is estimated to be in the order of 1 db.

Other possible disadvantages are that the array could cost more than single antennas because of the additional components in a phased array; and an increase in parts may also decrease the reliability figure. The higher manufacturing cost is there, and is expected because the array can do more. Lower reliability figure may not be significant because the array system uses only linear passive components.

On the other hand, other advantages are that the antenna system is complete in itself. It can be expanded by either going to a larger matrix network [3] or using the modular construction concept and stack the array up vertically.

8. RECOMMENDATIONS

This initial study has proven the feasibility of the concept of progressively phased circular array for antenna isolation. It is one of the few that are applicable to FAA communications..

It is thus strongly recommended that the study be extended to investigate the limitations of matrix feed networks, and effects of these limitations. Those limitations that should be investigated are loss, amplitude balance, phase precision, and isolation between phase progression modes.

REFERENCES

[1] R.F. Harrington, Field Computation By Moment Methods, The Macmillam Company, New York, 1968.

[2] J.D. Kraus, Antennas, McGraw-Hill Book Co., New York, 1950.

[3] B. Chiang, " A Foam Dielectric Matrix Fed Electronically Despun Circular Array ", 1970 IEEE GAP Intl. Symp. Digest, pp. 29-36, Ohio.

APPENDIX A

GENERALIZED ADMITTANCE MATRIX

This Fortran IV program calculates the generalized admittance matrix, and associated admittance and impedances.

Plots of the current distributions along dipoles can be obtained for all phase sequences. Necessary subroutines are included.

INPUT DATA:

EH	-	half height of dipoles (height of monopole).
NN	-	number of dipoles.
IPP	-	number of match points.
X(I)	-	X-coordinates.
Y(I)	-	Y-coordinates.
A	-	radius of dipole.
I	-	index of antenna number.

OUTPUT DATA:

X(I), Y(I)	-	defined above.
A	-	radius of dipole.
CS(K,L)	-	the generalized admittance matrix.
CSSIM(IJ,KJ)	-	short-circuited admittances and open-circuited impedances.

Assumptions are that the elements are thin, center-fed symmetrical dipoles. Surface resistance is negligible. Kernel is approximated. Moment method is used.

Some of the subroutines are the work of staff and students of University of Mississippi.

```
      IMPLICIT COMPLEX(C)
      COMPLEX CSN(40,40)
      COMPLEX DETERM,COEF(60),CV(64)
      COMPLEX*16 CS(40,40)
      COMPLEX CSSIM(10,10)
      DIMENSION AMAGI(240)
      COMMON X(5),Y(5),Z,A,AK,N,I
C     A IS RADIUS, EA IS HALF HEIGHT OF CENTER FED DIPOLE,
C     IPP IS NUMBER OF MATCH POINTS, NN IS NUMBER OF DIPOLES,
C     M + N ARE FIELD POINTS, M=MATCH PT, N=DIPOLE
C     I + IP ARE SOIRCE PTS, I=DIPOLE, IP=BASIS SET.
C     IF THE END MATCH POINT SERVES AS A SOURCE,IT IS ZERO, BECAUSE
C     OF THE BOUNDARY CONDITION WHICH STATES THAT THE CURRENT AT THE
C     END OF THE WIRE IS ZERO.  BUT IF THE END MATCH PT SERVES AS A
C     FIELD POINT, SINCE IT IS NUST A LOCATION FOR COLLECTION OF MAG.
C     VECTOR POTENTIAL, IT IS NOT NECESSARILU ZERO.
C
      EXTERNAL CKS
      MT1=25
      AK=6.2831853
C     READ IN  EH ,NN, IPP.
      READ(5,210)EH,NN,IPP
 210  FORMAT(F5.3,2I2)
      NNL1=NN-1
      ANNL1=NNL1
      ANNN=NN
      MM=IPP+1
      MT=NN*MM
      MLNN=MT-NN
C     READ IN COORDINATES OF DIPOLES.
      READ(5,7)(X(I),Y(I),I=1,5)
  7   FORMAT(2F8.4)
C     READ IN RADIUS OF ARRAY
      READ(5,2)A
  2   FORMAT(F8.6)
C     WRITE OUT COORDINATES OF DIPOLE.
      WRITE(6,5)
  5   FORMAT(10X,'COORDINATES OF DIPOLE')
      DO 9 I=1,NN
  9   WRITE(6,4)I,X(I),I,Y(I)
  4   FORMAT(5X ,'X',I1,'=',F10.4,5X,'Y',I1,'=',F10.4)
      WRITE(6,3)A
  3   FORMAT(5X,'RADIUS IS=',F9.6)
      ANN=IPP
      DZ=EH/ANN
      II=DZ/A*4.
C
C     INITIALIZE CS(M,N)
```

```
C
      DO 400 M=1,MT
      DO 400 N=1,MT
  400 CS(M,N)=CMPLX(0.,0.)
C     CALCULATE MATRIX ELEM BEGIN WITH THE FIRST HALF SECTION.
C
      DO 101 N=1,NN
      IP=1
      ZN=0.
      ZNP1=DZ/2.
      DO 104 M=1,MM
      AM=M
      Z=(AM-1.)*DZ
      DO 105 I=1,NN
      L=IP+(I-1)*IPP
      J=M+(N-1)*MM
      CALL CWEDF(CKS,ZN,ZNP1,II,DETERM)
      CS(J,L)=DETERM
  105 CONTINUE
  104 CONTINUE
C
C     CALCULATE FULL MATCH SECTION--MATRIX ELEM., WHERE I AT IS MADE 0.
C
      DO 101 IP=2,IPP
      AP=IP
      ZN=DZ*(AP-1.5)
      ZNP1=ZN+DZ
      DO 103 M=1,MM
      AM=M
      Z=(AM-1.)*DZ
      DO 102 I=1,NN
      L=IP+(I-1)*IPP
      J=M+(N-1)*MM
      CALL CWEDF(CKS,ZN,ZNP1,II,DETERM)
      CS(J,L)=DETERM
  102 CONTINUE
  103 CONTINUE
  101 CONTINUE
C
C     CALCULATE TERMS DUE TO COSINES ( EVEN SYMMETRY).
C
      DO 200 M=1,MM
      AM=M
      ZM=(AM-1.)*DZ
      DO 200 N=1,NN
      NINDX=M+(N-1)*MM
      RS=COS(AK*ZM)/30.
  200 CS(NINDX,NN*IPP+N)=CMPLX(0.,RS)
C
```

```
C     INVERT MATRIX
C
      CALL CDMIN1(MT,CS,40,CDETM)
      WRITE(6,989)CDETM
      WRITE(6,990)((CS(K,L),L=1,MT),K=1,MT1)
      DO 205 K=1,MT1
      DO 205 L=1,MT
 205  CSN(K,L)=CS(K,L)
      WRITE(6,1)(X(I),I=1,5)
      WRITE(6,1)(Y(I),I=1,5)
 1    FORMAT(5(2X,E12.6))
      WRITE(7,1005)((CSN(K,L),L=1,MT),K=1,MT1)
 1005 FORMAT(20A4)
C
C     CALCULATE THE DRIVING COLUMN MATRIX FOR THE FIRST ELEM.
C
      DO 300 M=1,MM
      AM=M
      Z=(AM-1.)*DZ
      RV=-SIN(AK*Z)/60.
  300 CV(M)=CMPLX(0.,RV)
C
C      CALCULATE SHORT CIRCUIT ADMITTTANCES
C
      DO 701 KJ=1,NN
      JK=KJ*MM-MM
      DO 701 IJ=1,NN
      J=(IJ-1)*IPP+1
      COEF(J)=CMPLX(0.,0.)
      DO 700 L=1,MM
      CHANGE=CS(J,JK+L)
  700 COFF(J)=COEF(J)+CHANGE    *CV(L)
  701 CSSIM(IJ,KJ)=COEF(J)
      WRITE(6,921)((IJ,KJ,CSSIM(IJ,KJ),KJ=1,NN),IJ=1,NN)
C
C     CALCULATE OPEN CIUCUIT IMPEDANCES
C
      CALL CMIN1(NN,CSSIM,10,DETERM)
      WRITE (6,991)DETERM
      WRITE(6,920)((IJ,KJ,CSSIM(IJ,KJ),KJ=1,NN),IJ=1,NN)
  920 FORMAT(4(' Z',2I1,' =',2E12.5))
  921 FORMAT(4(' Y',2I1,' =',2E12.5))
C
C     BEGIN PHASE SEQUENCES.
C           JJ=1 FOR 0,2 FOR 90,3 FOR 180,4 FOR 270.
      DO 500 JJ=1,NNL1
C
C     DRIVING MATRIX FOR SUBSEQUENT ELEMENTS.
C
```

```
      AJJ=JJ-1
      JJJ=AJJ
      DO 301 M=1,MM
C
C      DRIVING ELEMS OF CENTER ANTENNA IS THE SAME AS ANT =1.
C
      CV(NNL1*MM+M)=CV(M)
      DO 301 N=2,NNL1
      NL1=N-1
      ANL1=NL1
      ANGLE=AK/ANNL1*AJJ*ANL1
  301 CV(NL1*MM+M)=CV(M)*CEXP(CMPLX(0.,ANGLE))
      NCENT=NNL1*IPP+1
      COEF(1)=CMPLX(0.,0.)
      COEF(NCENT)=COEF(1)
      MTLMM=MT-MM
C
C FEED POINT I AT ELEM. =1 + THE CENTER ELEM. HAVE NO DRIVE FROM CENT.
C
      DO 331 L=1,MTLMM
      CHANGE=CS(1,L)
      COEF(1)=COEF(1)+CHANGE *CV(L)
      CHANG2=CS(NCENT,L)
  331 COEF(NCENT)=COEF(NCENT)+CHANG2      *CV(L)
      AMPLI=CABS(COEF(1))
      WRITE(6,910)COEF(1),AMPLI
      AMPLI=CABS(COEF(NCENT))
      WRITE(6,911)COEF(NCENT),AMPLI
      CZ12=COEF(NCENT)
      MT1=MTLMM+1
      COEF(NCENT)=CMPLX(0.,0.)
C
C     FEED PT. I AT THE CENTER ELEMENT WITH NO DRIVE FROM CIRC. ARRAY.
C
      DO 332 L=MT1,MT
  332 COEF(NCENT)=COEF(NCENT)+CS(NCENT,L)*CV(L)
      YL=.01
      CV2=-CZ12/(COEF(NCENT)+YL)
      AMPLI=CABS(COEF(NCENT))
      WRITE(6,912)COEF(NCENT),AMPLI
      AMPLI=CABS(CV2)
      WRITE(6,913)CV2,AMPLI,YL
      DO 333 L=MT1,MT
  333 CV(L)=CV(L)*CV2
C
C     CALCULATE COEFFICIENTS
C
      DO 320 J=1,MLNN
      COEF(J)=CMPLX(0.,0.)
```

```
      DO 321 L=1,MT
      CHANGE=CS(J,L)
  321 COEF(J)=COEF(J)+CHANGE *CV(L)
      AMAGI(J)=CABS(COFF(J))
      AMAGI(  MLNN+J)= REAL(COEF(J))
      AMAGI(2*MLNN+J)=AIMAG(COEF(J))
      AMAGI(3*MLNN+J)=0.
  320 WRITE(6,999)CCEF(J),AMAGI(J)
C
C     CALC INPUT IMPEDANCES AND ADMITTANCES
C
      DO 600 I=1,NN
      AIL1=I-1
      ANGLE=AK/ANNL1*AJJ*AIL1
      MMI=MM*I-MM+1
      IIPLI=(I-1)*IPP
      DETERM=COEF(IIPLI+1)/CEXP(CMPLX(0.,ANGLE))
      AMPLI=CABS(DETERM)
      WRITE(6,904)I,DFTERM,AMPLI
      DETERM=CMPLX(1.,0.)/DETERM
      AMPLI=1./AMPLI
  600 WRITE(6,903)I,DETERM,AMPLI
  500 CALL RPLOT(AMAGI,4,MLNN,1.,1.,JJJ)
  900 FORMAT(' I= ',I3,'    CS(',2I3,') = ',2E11.3)
  901 FORMAT(' IM= ',I3,' Z= ',E11.3)
  902 FORMAT(' IP,ZN,ZNP1,II,N,= ',I3,2E11.3,2I4)
  903 FORMAT(' Z(',I2,') =',3E14.8)
  904 FORMAT(' Y(',I2,') =',3E14.8)
  910 FORMAT(' Y11 =',2E14.8,'      MAGNITUDE =',E14.8)
  911 FORMAT(' Y12 =',2E14.8,'      MAGNITUDE =',E14.8)
  912 FORMAT(' Y22 =',2E14.8,'      MAGNITUDE =',E14.8)
  913 FORMAT(' V =',3E14.8,'      Y LOAD =',E14.8)
  989 FORMAT(46H INVERTED MATRIX WHCSE NORMALIZED DETERMINANT ,2D16.6)
  990 FORMAT(10D11.3)
  991 FORMAT(46H INVERTED MATRIX WHCSE NORMALIZED DETERMINANT ,2E16.6)
  999 FORMAT(3E16.6)
 1000 CONTINUE
      CALL EXIT
      END
      COMPLEX FUNCTICN CKS(ZP)
      COMMON X(5),Y(5),Z,A,AK,N,I
      RR=A**2+(X(I)-X(N))**2+(Y(I)-Y(N))**2
      R1=SQRT(RR+(Z+ZP)**2)
      R2=SQRT(RR+(Z-ZP)**2)
      RK=COS(AK*R1)/R1+COS(AK*R2)/R2
      AIK=-SIN(AK*R1)/R1-SIN(AK*R2)/R2
      CKS=CMPLX(RK,AIK)
       RETURN
       END
```

```
      SUBROUTINE CWEDF (CF,XL,XU,NX,CANS)                                CWEDF001
C                                                                               *
C        CWEDF IS A SUBROUTINE WHICH WILL NUMERICALLY INTEGRATE A USER          *
C        SUPPLIED FUNCTION BETWEEN SPECIFIED LIMITS.  (SINGLE PRECISION)        *
C          CF    - NAME OF FUNCTION SUBPROGRAM.  MUST BE LISTED IN AN           *
C                  EXTERNAL STATEMENT.                                          *
C          XL    - LOWER LIMIT OF INTEGRATION                                   *
C          XU    - UPPER LIMIT OF INTEGRATION                                   *
C          NX    - APPROXIMATE NUMBER OF NODES AT WHICH TO EVALUATE FUNCTION    *
C          CANS  - RESULT OF INTEGRATION                                        *
C             PREPARED BY MICHAEL G. HARRISON  E.E. DEPT    JUNE 22, 1972       *
C                                                                               *
      IMPLICIT COMPLEX*8 (C)                                             CWEDF002
      REAL*8 DXDX,XX                                                     CWEDF003
      REAL*4 CW(6)/82.,216.,27.,272.,27.,216./                           CWEDF004
      IF(NX.LE.0) GO TO 900                                              CWEDF005
      N=((NX+4)/6)*6+1                                                   CWEDF006
      DX=(XU-XL)/FLOAT(N-1)                                              CWEDF007
      DXDX=DBLE(DX)                                                      CWEDF008
      NWIX=N/6                                                           CWEDF009
      X=XL                                                               CWEDF010
      CANS=-CF(X)*41.0                                                   CWEDF011
      DO 800 MX=1,NWIX                                                   CWEDF012
      DO 700 KX=1,6                                                      CWEDF013
      CANS=CANS+CW(KX)*CF(X)                                             CWEDF014
      XX=DBLE(X)                                                         CWEDF015
      X=SNGL(XX+DXDX)                                                    CWEDF016
  700 CONTINUE                                                           CWEDF017
  800 CONTINUE                                                           CWEDF018
      CANS=(CANS+41.0*CF(X))*DX/140.0                                    CWEDF019
      RETURN                                                             CWEDF020
  900 WRITE(6,901) N                                                     CWEDF021
  901 FORMAT('0ERROR IN CALLING PARAMETER *****  N = ',I5,'  *****'//)   CWEDF022
      RETURN                                                             CWEDF023
      END                                                                CWEDF024
C     SUBROUTINE RPLCT
C
C     PURPOSE
C        PLOT SEVERAL DEPENDENT VARIABLES AGAINST ONE INDEPENDENT
C        VARIABLE
C
C     USAGE
C        CALL RPLCT (A,NC,NR,X1,XI,INO)
C
C     DESCRIPTION OF PARAMETERS
C        A    - THE ARRAY TO BE PLOTTED. EACH COLUMN CONTAINS A
C               VARIABLE TO BE PLOTTED
C        NC   - THE NUMBER OF COLUMNS IN A
C        NR   - THE NUMBER OF ROWS IN A
```

```
C        X1   - THE FIRST VALUE OF THE INDEPENDENT VARIABLE
C        XI   - INCREMENT OF THE INDEPENDENT VARIABLE
C        INO - CHART NUMBER (3DIGITS MAXIMUM)
C
C     REMARKS
C        NONE
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     ..................................................................
C
      SUBROUTINE RPLOT(A,NC,NR,XX1,XI,INO)
C
      REAL MIN,MAX,A(1)
      INTEGER IR(12),LINE(101),BLK
      X1=XX1
C
1     FORMAT(18H1    CHART NUMBER ,I3,/,1H0,E15.6,71X,E15.6,13X,7HX VALU
     .E,//)
2     FORMAT(1H ,101A1,8X,E15.6)
3     FORMAT(1H0,80X,I6,15H POINTS PLOTTED)
C
C     INITIALIZE VARIABLES
C
      BLK=2**30
      IR(1)=2**30+2**27+2**25+2**24                                   PLOT '.'
      IR(2)=2**30+2**29+2**27+2**25+2**24                             PLOT ','
      IR(3)=2**30+2**27+2**26+2**25                                   PLOT '+'
      IR(4)=2**30+2**28+2**27+2**26                                   PLOT '*'
      IR(5)=2**30+2**27+2**25+2**24                                   PLOT '1'
      IR(6)=2**30+2**29+2**27+2**25+2**24                             PLOT '2'
      IR(7)=2**30+2**27+2**26+2**25                                   PLOT '3
      IR(8)=2**30+2**28+2**27+2**26                                   PLOT '4'
      IR(9)=2**30+2**27+2**25+2**24                                   PLOT '5'
      IR(10)=2**30+2**29+2**27+2**25+2**24                            PLOT '6'
      IR(11)=2**30+2**27+2**26+2**25                                  PLOT '7'
      IR(12)=2**30+2**28+2**27+2**26                                  PLOT '8'
C
      DO 4 I=1,101
      LINE(I)=BLK
4     CONTINUE
C
C     LOCATE MIN AND MAX VALUES
C
      N=NC*NR
      MIN=A(1)
      MAX=A(1)
C
```

```
      DO 8 I=2,N
      IF(A(I)-MIN) 5,6,6
5     MIN=A(I)
6     CONTINUE
      IF(A(I)-MAX) 8,8,7
7     MAX=A(I)
8     CONTINUE
C
C     FOR SINGLE VALUED ARRAY SET LIMITS
C
      IF( AX-MIN) 9,9,10
9     MAX=A(1)+1.0
      MIN=A(1)-1.0
10    CONTINUE
C
      WRITE(6,1) INC,MIN,MAX
C
C     BEGIN PLOT LOOP
C
      DO 15 I=1,NR
      DO 14 K=1,NC
      IF(K-1) 11,11,12
11    KSA=I
      GO TO 13
12    KSA=KSA+NR
13    CONTINUE
      KPNT=(A(KSA)-MIN)/(MAX-MIN)*100.0+1.5
      LINE(KPNT)=IR(K)
14    CONTINUE
      WRITE(6,2) LINE,X1
      X1=X1+XI
      DO 15 L=1,101
      LINE(L)=BLK
15    CONTINUE
C
      WRITE(6,3) NR
C
      RETURN
      END
      SUBROUTINE CMINI (N,A,NDIM,DETERM)                                CMN10010
C                                                                              *
C        CMINI IS A SUBROUTINE WHICH WILL ACCEPT A SINGLE PRECISION COMPLEX    *
C        MATRIX AND RETURN THE INVERSE OF THE MATRIX IN ITS PLACE.    THE      *
C        SUBROUTINE WILL ALSO COMPUTE THE NORMALIZED DETERMINANT OF THE MATRIX.*
C              N     - THE ORDER OF THE MATRIX TO BE INVERTED                  *
C              A     - COMPLEX DOUBLE PRECISION INPUT MATRIX (DESTROYED)       *
C                      THE INVERSE OF A IS RETURNED IN ITS PLACE.              *
C              NDIM  - THE SIZE TO WHICH A IS DIMENSIONED IN THE CALLING PROGRAM *
C              DETERM THE NORMALIZED DETERMINANT WHICH IS CALCULATED BY THE    *
```

```
C            DETERM - THE NORMALIZED DETERMINANT OF A WHICH IS RETURNED       *
C         PREPARED BY MICHAEL G. HARRISON E.E. DEPT      JUNE 23, 1972       *
C                                                                            *
      COMPLEX A(NDIM,NDIM),PIVOT(100),AMAX,T,SWAP,DETERM,U              CMN10020
      INTEGER*4 IPIVOT(100),INDEX(100,2)                                 CMN10030
      REAL   TEMP,ALPHA(100)                                            CMN10040
C                                                                       CMN10050
C     INITIALIZATION                                                    CMN10060
C                                                                       CMN10070
      DETERM =  CMPLX(1.0,0.0)                                          CMN10080
      DO 20 J=1,N                                                       CMN10090
      ALPHA(J)=0.0D0                                                    CMN10100
      DO 10 I=1,N                                                       CMN10110
   10 ALPHA(J)=ALPHA(J)+A(J,I)* CONJG(A(J,I))                           CMN10120
      ALPHA(J)= SQRT(ALPHA(J))                                          CMN10130
   20 IPIVOT(J)=0                                                       CMN10140
      DO 600 I=1,N                                                      CMN10150
C                                                                       CMN10160
C     SEARCH FOR PIVOT ELEMENT                                          CMN10170
C                                                                       CMN10180
      AMAX=CMPLX(0.0,0.0)                                               CMN10190
      DO 105 J=1,N                                                      CMN10200
      IF (IPIVOT(J)-1) 60, 105, 60                                      CMN10210
   60 DO 100 K=1,N                                                      CMN10220
      IF (IPIVOT(K)-1) 80, 100, 740                                     CMN10230
   80 TEMP=AMAX* CONJG(AMAX)-A(J,K)* CONJG(A(J,K))                      CMN10240
      IF(TEMP)85,85,100                                                 CMN10250
   85 IROW=J                                                            CMN10260
      ICOLUM=K                                                          CMN10270
      AMAX=A(J,K)                                                       CMN10280
  100 CONTINUE                                                          CMN10290
  105 CONTINUE                                                          CMN10300
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1                                   CMN10310
C                                                                       CMN10320
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL                 CMN10330
C                                                                       CMN10340
      IF (IROW-ICOLUM) 140, 260, 140                                    CMN10350
  140 DETERM=-DETERM                                                    CMN10360
      DO 200 L=1,N                                                      CMN10370
      SWAP=A(IROW,L)                                                    CMN10380
      A(IROW,L)=A(ICOLUM,L)                                             CMN10390
  200 A(ICOLUM,L)=SWAP                                                  CMN10400
      SWAP=ALPHA(IROW)                                                  CMN10410
      ALPHA(IROW)=ALPHA(ICOLUM)                                         CMN10420
      ALPHA(ICOLUM)=SWAP                                                CMN10430
  260 INDEX(I,1)=IROW                                                   CMN10440
      INDEX(I,2)=ICOLUM                                                 CMN10450
      PIVOT(I)=A(ICOLUM,ICOLUM)                                         CMN10460
      U = PIVOT(I)                                                      CMN10470
```

```
      DETERM = DETERM*U                                                 CMN10480
      DETERM=DETERM/ALPHA(ICOLUM)                                       CMN10490
      TEMP=PIVOT(I)* CONJG(PIVOT(I))                                    CMN10500
      IF(TEMP)330,720,330                                               CMN10510
C                                                                       CMN10520
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT                                 CMN10530
C                                                                       CMN10540
  330 A(ICOLUM,ICOLUM) =  CMPLX(1.0,0.0)                                CMN10550
      DO 350 L=1,N                                                      CMN10560
      U = PIVOT(I)                                                      CMN10570
  350 A(ICOLUM,L) = A(ICOLUM,L)/U                                       CMN10580
C                                                                       CMN10590
C     REDUCE NON-PIVOT ROWS                                             CMN10600
C                                                                       CMN10610
 380  DO 550 L1=1,N                                                     CMN10620
      IF(L1-ICOLUM) 400, 550, 400                                       CMN10630
  400 T=A(L1,ICOLUM)                                                    CMN10640
      A(L1,ICOLUM)= CMPLX(0.0,0.0)                                      CMN10650
      DO 450 L=1,N                                                      CMN10660
      U = A(ICOLUM,L)                                                   CMN10670
  450 A(L1,L) = A(L1,L)-U*T                                             CMN10680
  550 CONTINUE                                                          CMN10690
 600  CONTINUE                                                          CMN10700
C                                                                       CMN10710
C     INTERCHANGE COLUMNS                                               CMN10720
C                                                                       CMN10730
 620  DO 710 I=1,N                                                      CMN10740
      L=N+1-I                                                           CMN10750
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630                          CMN10760
  630 JROW=INDEX(L,1)                                                   CMN10770
      JCOLUM=INDEX(L,2)                                                 CMN10780
      DO 705 K=1,N                                                      CMN10790
      SWAP=A(K,JROW)                                                    CMN10800
      A(K,JROW)=A(K,JCOLUM)                                             CMN10810
      A(K,JCOLUM)=SWAP                                                  CMN10820
  705 CONTINUE                                                          CMN10830
  710 CONTINUE                                                          CMN10840
      RETURN                                                            CMN10850
 720  WRITE(6,730)                                                      CMN10860
  730 FORMAT(20H  MATRIX IS SINGULAR)                                   CMN10870
  740 RETURN                                                            CMN10880
      END                                                               CMN10890
      SUBROUTINE COMINI (N,A,NDIM,DETERM)                               CDM10010
C                                                                              *
C        COMINI IS A SUBROUTINE WHICH WILL ACCEPT A DOUBLE PRECISION COMPLEX   *
C        MATRIX AND RETURN THE INVERSE OF THE MATRIX IN ITS PLACE.    THE      *
C        SUBROUTINE WILL ALSO COMPUTE THE NORMALIZED DETERMINANT OF THE MATRIX.*
C             N    - THE ORDER OF THE MATRIX TO BE INVERTED                    *
C             A    - COMPLEX DOUBLE PRECISION INPUT MATRIX (DESTROYED)         *
```

```
C                    THE INVERSE OF A IS RETURNED IN ITS PLACE.                  *
C             NDIM - THE SIZE TO WHICH A IS DIMENSIONED IN THE CALLING PROGRAM *
C             DETERM - THE NORMALIZED DETERMINANT OF A WHICH IS RETURNED        *
C        PREPARED BY MICHAEL G. HARRISON E.E. DEPT      JUNE 23, 1972           *
C                                                                               *
      COMPLEX*16 A(NDIM,NDIM),PIVOT(100),AMAX,T,SWAP,DETERM,U
      COMPLEX*16 CMPLX,DCONJG,CDINV,CDXXXX
      INTEGER*4 IPIVOT(100),INDEX(100,2)
      REAL*8 ALPHA(100),TEMP
C
C     INITIALIZATION
C
      DETERM = DCMPLX(1.D+0,0.D+0)
      DO 20 J=1,N
      ALPHA(J)=0.D00
      DO 10 I=1,N
   10 ALPHA(J)=ALPHA(J)+[illegible],I)*DCONJG(A(J,I))
      ALPHA(J)=DSQRT(ALPHA(J))
   20 IPIVOT(J)=0
      DO 600 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
      AMAX = DCMPLX(0.D+0,0.D+0)
      DO 105 J=1,N
      IF (IPIVOT(J)-1) 60, 105, 60
   60 DO 100 K=1,N
      IF (IPIVOT(K)-1) 80, 100, 740
80    TEMP=AMAX*DCONJG(AMAX)-A(J,K)*DCONJG(A(J,K))
      IF(TEMP)85,85,100
   85 IROW=J
      ICOLUM=K
      AMAX=A(J,K)
  100 CONTINUE
  105 CONTINUE
      IPIVOT(ICOLUM)=IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW-ICOLUM) 140, 260, 140
  140 DETERM=-DETERM
      DO 200 L=1,N
      SWAP=A(IROW,L)
      A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      SWAP=ALPHA(IROW)
      ALPHA(IROW)=ALPHA(ICOLUM)
      ALPHA(ICOLUM)=SWAP
  260 INDEX(I,1)=IROW
```

```
      INDEX(I,2)=ICOLUM
      PIVOT(I)=A(ICOLUM,ICOLUM)
      U = PIVOT(I)
      DETERM = DETERM*U
      DETERM=DETERM/ALPHA(ICOLUM)
      TEMP=PIVOT(I)*DCONJG(PIVOT(I))
      IF(TEMP)330,720,330
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
  330 A(ICOLUM,ICOLUM) = DCMPLX(1.D+0,0.D+0)
      DO 350 L=1,N
      U = PIVOT(I)
  350 A(ICOLUM,L) = A(ICOLUM,L)*CDXXXX(U)
C
C     REDUCE NON-PIVOT ROWS
C
 380  DO 550 L1=1,N
      IF(L1-ICOLUM) 400, 550, 400
  400 T=A(L1,ICOLUM)
      A(L1,ICOLUM)=DCMPLX(0.D+0,0.D+0)
      DO 450 L=1,N
      U = A(ICOLUM,L)
  450 A(L1,L) = A(L1,L)-U*T
  550 CONTINUE
600   CONTINUE
C
C     INTERCHANGE COLUMNS
C
 620  DO 710 I=1,N
      L=N+1-I
      IF (INDEX(L,1)-INDEX(L,2)) 630, 710, 630
  630 JROW=INDEX(L,1)
      JCOLUM=INDEX(L,2)
      DO 705 K=1,N
      SWAP=A(K,JROW)
      A(K,JROW)=A(K,JCOLUM)
      A(K,JCOLUM)=SWAP
  705 CONTINUE
  710 CONTINUE
      RETURN
 720  WRITE(6,730)
  730 FORMAT(20H  MATRIX IS SINGULAR)
  740 RETURN
      END
      COMPLEX FUNCTION CDXXXX*16(A)
      COMPLEX*16 A,DCMPLX
      REAL*8 AR,AI,ARINV,AIINV,DABS
      AR=A
```

```
      AI=-(0.D+0,1.D+0)*A
      IF(DABS(AR) .LE. 1.D-30)AR=0.D+0
      IF(DABS(AI) .LE. 1.D-30)AI=0.D+0
      ARINV=AR/(AR*AR+AI*AI)
      AIINV=-AI/(AR*AR+AI*AI)
      CDXXXX=DCMPLX(ARINV,AIINV)
      RETURN
      END
```

APPENDIX B

MOMENT METHOD DATA REDUCTION BY SEQUENCE FUNCTIONS

ABSTRACT

The generalized admittance matrix of wire antennas obtained by applying the moment method is usually large, and cumbersome to handle. In order to facilitate repeated calculations to find field and circuit quantities from the matrix, sequence functions are used. The approach avoids the necessity to store the entire admittance matrix and also reduces subsequent computing efforts. It supplies current distributions and field patterns of each sequence as an intermediate step, and thus provides otherwise unobtainable insight into the performance of an antenna system, especially that of a conformal array.

MOMENT METHOD DATA REDUCTION BY SEQUENCE FUNCTIONS*

Bing Chiang, Howard University, Washington, D.C.

In applying the moment method for solutions of wire antennas, the labor is in calculating the impedance matrix, and in matrix inversion to obtain the admittance matrix. Matrices encountered in this solution method are generally very large. To calculate field and circuit quantities directly from these matrices is an equally laborious task due to sheer bulk of data. In order to reduce data handling, the approach of using generalized sequence functions is suggested.

Sequence functions have been used by King, Mack and Sandler[1] to analyse symmetrical circular arrays. However, antennas and arrays often have arbitrary shapes and sizes and are often unsymmetrical; therefore, a generalized sequence function theory is needed. The theory, developed below, is valid wherever super-position holds.

Current distributions on wire antennas are shown in the moment method as[2]

$$[I_n] = [Y_{nm}]\ [V_m] \qquad (1)$$

where, I is the current distribution along the antenna,
Y is the generalized admittance matrix,
V is the generalized voltage,
n is the index of the basis set which goes from 1 to N,

and lastly,
m is the index of the testing set which goes from 1 to M.

The size of the generalized admittance matrix is thus NxM, where $N \leq M$.

If there are P ports in the antenna system, then the matrix size can be shown to reduce to NxP when sequence function is used. The saving is thus M/P fold.

Formulation of the sequence function is based on that any applied voltage V_p in a set of P voltages can be specified by a linear combination of P sequence voltages:

$$V_p = \sum_{m=0}^{P-1} A^{(m)} \exp\ [j2\pi(p-1)m/P] \qquad (2)$$

where $A^{(m)}$ is the complex coefficient of the mth

sequence. Shelton-Butler matrix [3] is the parallel physical system that performs sequence generation and summation as shown in equation (2). An array fed by such a matrix is shown in Figure 1. In this figure, only the $\pi/2$ sequence is excited, so that the progressive phase going from port to port can be illustrated clearly. The random orientation of antennas and location of antenna ports depict the general applicability of the theory. When all matrix input ports are excited simultaneously, the voltage at the antenna port is thus the v_p shown in equation (2). Any set of v_p can be completely specified by a set of $A^{(m)}$.

Using the moment method, current distributions on antennas can be calculated from the generalized admittance matrix. Once the current distribution for each voltage sequence is calculated, the response of any arbitrary set of voltages v_p can be found by scaling and superpositioning . And because v_p can be synthesized by scaling and superpositioning, it is obvious then, the only information worthy of storage is the current distributions caused by a set of normalized sequence voltages. There are N data points for each set of distributions, and there are P sequences. Total data points are thus equal to N x P, which is a reduction from N x M.

In applying sequence voltages, data stored are not necessarily limited to current distributions. If one so wishes, he may store the complex field pattern for each mode instead, and obtain the desired field pattern by again scaling and superpositioning.

As an example, a circular array of dipoles as shown in Figure 2 with an array radius of 0.3 wavelength, and a passive element near the center was analyzed. For simplicity of analysis all elements were made equal. They have a wire diameter of 0.025 wavelength. The passive element was displaced from the array center by .03 wavelength, and was loaded by a 100 ohm resistor. Each half of a dipole was divided into 5 pulses, with end-sections having half the width.

Field and phase patterns of each sequence were calculated. They are plotted in Figure 3. Let it be assumed that the desired feed voltages are those shown in Table 1. Since the sequences form an orthogonal set, the desired sequence voltages can be found, and are listed in Table 2. The resultant pattern is obtained by scaling and superpositioning the patterns shown in Figure 3 and is plotted in Figure 4.

ACKNOWLEDGEMENT

The author wishes to thank Dr. Chalmers M. Butler of the University of Mississippi for his invaluable discussion on the topic of moment method.

References

1. R.W.P. King, R.B. Mack, and S.S. Sandler, Arrays of Cylindrical Dipoles, bridge University Press, London, 1968.

2. R.F. Harrington, Field Computation by Moment Methods, Macmillan Co., New York, 1968.

3. B. Chiang, R. Yaminy, and R. Jackson, "A Foam Dielectric Matrix Fed Electronically Despun Circular Array", GAP International Symposium, pp. 29-36, Columbus, Ohio, September, 1970.

*This work was supported by FAA, Contract DOT-FA-73 WA-3156.

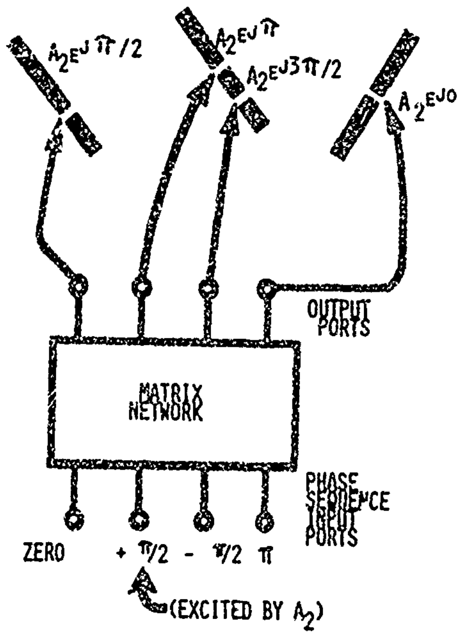


FIGURE 1: MATRIX FED ARRAY SHOWING ANTENNA PORT PHASE PROGRESSION WHEN THE + $\pi/2$ PORT IS EXCITED.

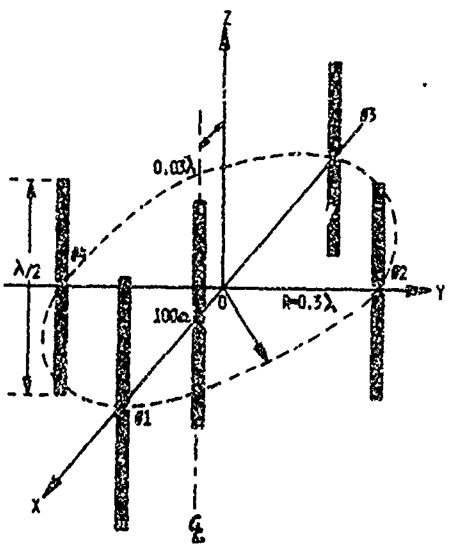


FIGURE 2: FOUR ELEMENT CIRCULAR ARRAY WITH A LOADED DIPOLE NEAR CENTER.

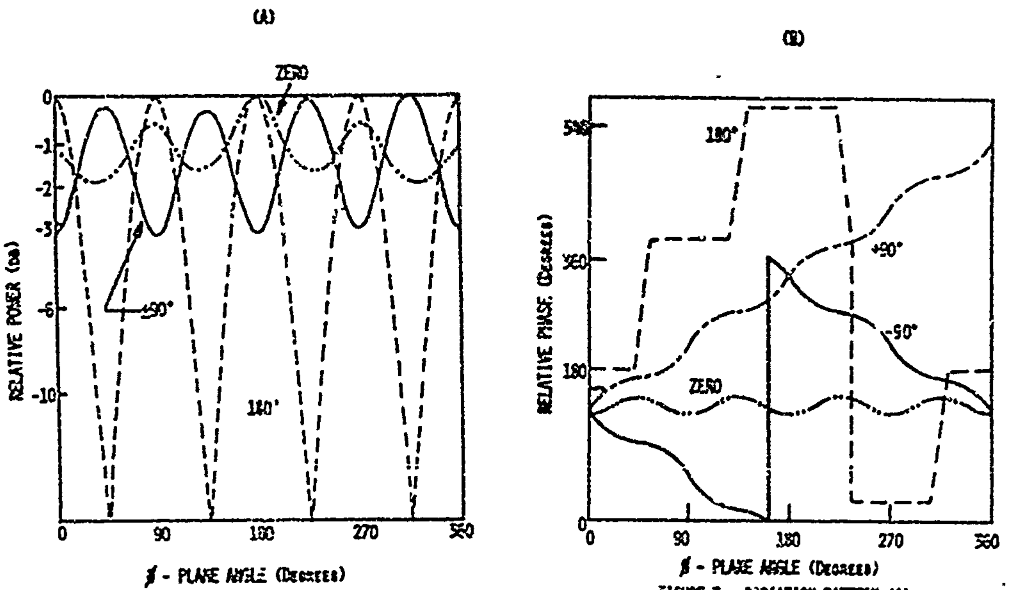


FIGURE 3. RADIATION PATTERN (A) AND PHASE PATTERN (B) OF THE ARRAY SHOWN IN FIGURE 2. PHASE PROGRESSIONS ARE AS INDICATED.

TABLE I

DESIRED DIPOLE FEED POINT VOLTAGES TO FORM A MAIN BEAM IN THE DIRECTION OF +X.

V_1 =	3.4827-j1.1008	V_2 =	0.5251+j0.8802
V_3 =	-.5129-j0.6596	V_4 =	0.5251+j0.8802

TABLE II

THE NECESSARY SEQUENCE VOLTAGES

A_1 =	1	A_2 =	$1\angle -6.3^\circ$
A_3 =	$1\angle -6.3^\circ$	A_4 =	$1\angle -52.7^\circ$

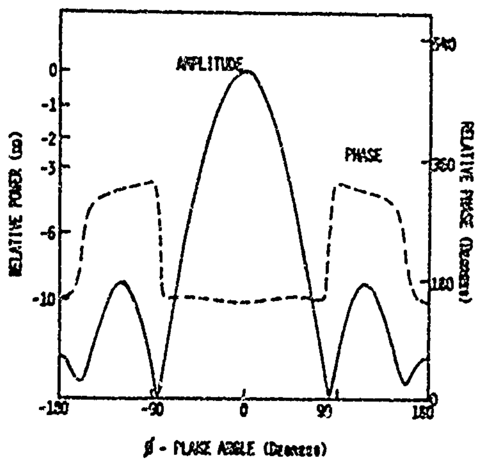


FIGURE 4. RADIATION AND PHASE PATTERNS OF THE ARRAY WHEN ALL PROGRESSIONS ARE EXCITED PER TABLE II.

APPENDIX C

MISCELANEOUS COMPUTER PROGRAMS

INPUT DATA:

IPP	-	number of match points.
NM	-	number of dipoles.
EH	-	half height of dipoles.
CS(I,J,K)	-	inverted short-circuited matrix.
I	-	row indexing.
J	-	column indexing.
K	-	real or imaginary indexing.
X(L)	-	value along the X-coordinate.
Y(L)	-	value along the Y-coordinate.
L	-	dipole number indexing.
RAD	-	radius of dipoles.
PERRX	-	percentage error displacement in X(5).
PERRY	-	percentage error displacement in Y(5).

OUTPUT DATA:

X(L)	-	X-coordinate of dipoles.
Y(L)	-	Y-coordinate of dipoles.
RAD	-	radius of dipole.
PERRX	-	percentage error displacement in X(5).
PERRY	-	percentage error displacement in Y(5).
CSSIM(IJ,KJ,N)	-	short-circuited admittance matrix.
IJ	-	row indexing.
KJ	-	column indexing.
N	-	real and imaginary indexing.
VL(I,K)	-	phase voltage of center louded dipole.
I	-	row indexing.
K	-	column indexing.
TCOEF(I,N,K)	-	total current distribution, where I is row indexing, N is column indexing, and K is real or imaginary indexing.
AMPLI(I,N)	-	amplitude of current with the same indexing as TCOEF.
TCOEF	-	is also punched on cards.

```
C     PROGRAM TO CALCULATE  SHORT CIRCUIT ADMITTANCE
C     PHASE VOLTAGE OF CENTER LOADED DIPOLE
C     AND TOTAL CURRENT DISTRIBUTION OF ARRAY.
      DIMENSION Y(5,5),DIV(2),YCOEF(5,2),VL(5,2)
      DIMENSION TRM(5),YTRM(5),P(1,2)
      DIMENSION ANG2(5),ANG3(5),ANG4(5),E(4,5,2)
      DIMENSION CS(25,30,2),CV(10),COEF(10,10,2),CSSIM(10,10,2)
      DIMENSION CI(5,4,2),CCOEF(5,4,2)
      DIMENSION TCI(30,4,2),TCOEF(25,4,2)
      DIMENSION AV(2)
      DIMENSION AMPLI(25,4)
      DIMENSION X(5),Y1(5)
C
      IPP=5
      NN=5
      EH=.25
      AK=6.2831853
      MM=IPP+1
      ANN=NN
      DZ=EH/ANN
      IN=2
      IOUT=5
C     READ IN INVERTED MATRIX
      READ(2,101)(((CS(I,J,K),K=1,2),J=1,30),I=1,25)
  101 FORMAT(20A4)
      DO 102 I=1,25
      DO 102 J=1,30
      DO 102 K=1,2
  102 CS(I,J,K)=A4CVT(CS(I,J,K))
C
C     READ IN COORDINATES OF DIPOLES
      READ(2,250)(X(I),Y1(I),I=1,NN)
  250 FORMAT(2F6.4)
C     PRINT COORDINATES OF DIPOLES
      WRITE(5,254)
  254 FORMAT(1H1,6X,'COORDINATES OF DIPOLES')
      DO 255 I=1,NN
  255 WRITE(5,256)I,X(I),I,Y1(I)
  256 FORMAT(5X,'X',I1,'=',F7.4,3X,'Y',I1,'=',F7.4)
C     READ IN RADIUS OF DIPOLE
      READ(2,259)RAD
  259 FORMAT(F6.4)
      WRITE(5,260)RAD
  260 FORMAT(6X,'RADIUS OF DIPOLE IS',F7.4)
      READ(2,257)PERRX,PERRY
  257 FORMAT(2F7.4)
      WRITE(5,258)PERRX,PERRY
  258 FORMAT(5X,'PERCENTAGE ERROR DISPLACEMENT IN X5 AND Y5',/15X,'X5='
     1,F6.4,'Y5=',F8.4)
```

```
C       DRIVING COLUMN MATRIX
        DO 300 M=1,MN
        AM=M
        Z=(AM-1.)*DZ
        RV=-SIN(AK*Z)/60.
  300   CV(M)=RV
C
C       CALCULATE SHORT CKT ADMITTANCES
C
        DO 701 KJ=1,NN
        JK=KJ*MM-MM
        DO 701 IJ=1,NN
        J=(IJ-1)*IPP+1
        DO 702 K=1,2
  702   COEF(J,KJ,K)=0.0
        DO 700 L=1,MM
        DO 700 K=1,2
        LL=JK+L
        CHANG=CS(J,LL,K)
  700   COEF(J,KJ,K)=COEF(J,KJ,K)+CHANG*CV(L)
        DO 701IK=1,2
        IIK=-IK
        N=IIK+3
        IF(N-1)20,25,20
  25    CSSIM(IJ,KJ,N)=-COEF(J,KJ,IK)
        GO TO 701
  20    CSSIM(IJ,KJ,N)=COEF(J,KJ,IK)
  701   CONTINUE
        WRITE(5,111)
  111   FORMAT(1H1,6X,'SHORT CIRCUIT ADMITTANCE')
        WRITE(IOUT,921)(((CSSIM(IJ,KJ,N),N=1,2),KJ=1,5),IJ=1,5)
  921   FORMAT(5X,E13.6,5X,E13.6)
        ANG1=0.0
        RAD=3.141593/180.
        DEG=ANG1*RAD
C       CALCULATE DRIVING VOLTAGE FOR '0' PHASE
        I=1
        DO 100 K=1,2
        DO 105 J=1,4
        IF(K-1)5,3,5
  3     E(I,J,K)=COS(DEG)
        GO TO 105
  5     E(I,J,K)=SIN(DEG)
  105   CONTINUE
  100   CONTINUE
C       CALCULATE DRIVING VOLTAGE FOR '+90' PHASE
        IN=0
        DO 110 N=90,360,90
        IN=IN+1
```

```
      THETA=N-90
      DEG=THETA*RAD
      ANG2(IN)=DEG
 110  CONTINUE
      I=2
      DO 115 K=1,2
      DO 120 J=1,4
      IF(K-1)9,7,9
 7    A=ANG2(J)
      E(I,J,K)=COS(A)
      GO TO 120
 9    B=ANG2(J)
      E(I,J,K)=SIN(B)
 120  CONTINUE
 115  CONTINUE
C     CALCULATE DRIVING VOLTAGE FOR '-90' PHASE
      IN=0
      DO 125 N=90,360,90
      IN=IN+1
      THETA=90-N
      DEG=THETA*RAD
      ANG3(IN)=DEG
 125  CONTINUE
      I=3
      DO 130 K=1,2
      DO 135 J=1,4
      IF(K-1)11,13,11
 13   A=ANG3(J)
      E(I,J,K)=COS(A)
      GO TO 135
 11   B=ANG3(J)
      E(I,J,K)=SIN(B)
 135  CONTINUE
 130  CONTINUE
C     CALCULATE DRIVING VOLTAGE FOR '180' PHASE
      IN=0
      DO 140 NT=1,2
      DO 145 N=180,360,180
      IN=IN+1
      THETA=N-180
      DEG=THETA*RAD
      ANG4(IN)=DEG
 145  CONTINUE
 140  CONTINUE
      I=4
      DO 150 K=1,2
      DO 155 J=1,4
      IF(K-1)15,17,15
 17   A=ANG4(J)
```

```
      E(I,J,K)=COS(A)
      GO TO 155
 15   B=ANG4(J)
      E(I,J,K)=SIN(B)
 155  CONTINUE
 150  CONTINUE
C     LOAD ADMITTANCE
      YREAL=-0.01
      YIMAG=0.0
      IJ=5
      DO 160 KJ=1,5
      DO 160 K=1,2
 160  Y(KJ,K)=CSSIM(IJ,KJ,K)
      CREAL=Y(5,1)
      CIMAG=Y(5,2)
      DIV(1)=YREAL-CREAL
      DIV(2)=-(YIMAG-CIMAG)
      D=DIV(1)**2+DIV(2)**2
      DO 170 I=1,4
      DO 180 K=1,2
 180  YCOEF(I,K)=0.0
      DO 175 J=1,4
      DO 185 K=1,2
      TRM(K)=E(I,J,K)
 185  YTRM(K)=Y(J,K)
      CALL CMULT(TRM,YTRM,P)
      DO 190 K=1,2
      YINCR=P(1,K)
 190  YCOEF(I,K)=YCOEF(I,K)+YINCR
 175  CONTINUE
 170  CONTINUE
      DO 200 I=1,4
      DO 205 K=1,2
      TRM(K)=YCOEF(I,K)
 205  YTRM(K)=DIV(K)
      CALL CMULT(TRM,YTRM,P)
      DO 210 K=1,2
 210  VL(I,K)=P(1,K)/D
 200  CONTINUE
      WRITE(5,1000)
 1000 FORMAT(1H1,6X,'PHASE VOLTAGE OF CENTER LOADED DIPOLE')
      WRITE(5,112)((VL(I,K),K=1,2),I=1,4)
 112  FORMAT(2(5X,E13.6))
      J=5
      DO 333 I=1,4
      DO 333 K=1,2
 333  E(I,J,K)=VL(I,K)
      DO 555I=1,4
      DO 555J=1,5
```

```
      DO 555 K=1,2
  555 CI(J,I,K)=E(I,J,K)
      WRITE(5,557)
  557 FORMAT(///,15X,'CI(J,I,K) MATRIX')
      WRITE(5,556)(((CI(J,I,K),K=1,2),I=1,4),J=1,5)
  556 FORMAT(8(2X,F10.4))
      DO 320 I=1,5
      DO 330 N=1,4
      DO 310 K=1,2
  310 CCOEF(I,N,K)=0.0
      DO 360 J=1,5
      DO 340 K=1,2
      TRM(K)=CSSIM(I,J,K)
  340 YTRM(K)=CI(J,N,K)
      CALL CMULT(TRM,YTRM,P)
      DO 350 K=1,2
  350 CCOEF(I,N,K)=CCOEF(I,N,K)+P(1,K)
  360 CONTINUE
  330 CONTINUE
  320 CONTINUE
      WRITE(5,558)
  558 FORMAT(///,15X,'CCOEF(I,N,K) MATRIX')
      WRITE(5,556)(((CCOEF(I,N,K),K=1,2),N=1,4),I=1,5)
C     CALCULATE TOTAL CURRENT DISTRIBUTION
C     CALCULATE TOTAL VOLTAGE PER PHASE
      DO 407 J=1,4
      DO 400 I=1,5
      DO 425 M=1,MM
      II=(I-1)*6+M
      AM=M
      Z=(AM-1.)*DZ
      AV(1)=0.0
      AV(2)=-SIN(AK*Z)/60.
      DO 426 K=1,2
      TRM(K)=CI(I,J,K)
  426 YTRM(K)=AV(K)
      CALL CMULT(TRM,YTRM,P)
      DO 427 K=1,2
  427 TCI(II,J,K)=P(1,K)
  425 CONTINUE
  400 CONTINUE
  407 CONTINUE
      DO 450 I=1,25
      DO 450 N=1,4
      DO 430 K=1,2
  430 TCCEF(I,N,K)=0.0
      DO 450 J=1,30
      DO 455 K=1,2
      TRM(K)=CS(I,J,K)
```

```
455   YTRM(K)=TCI(J,N,K)
      CALL CMULT(TRM,YTRM,P)
      DO 450 K=1,2
450   TCOEF(I,N,K)=TCOEF(I,N,K)+P(1,K)
      WRITE(5,666)
666   FORMAT(///,40X,'TOTAL CURRENT DISTRIBUTION')
      DO 510 N=1,4
      DO 510 I=1,25
510   AMPLI(I,N)=SQRT((TCOEF(I,N,1))**2+(TCOEF(I,N,2))**2)
      WRITE(5,997)(((TCOEF(I,N,K),K=1,2),AMPLI(I,N),N=1,4),I=1,25)
997   FORMAT(6(1X,E13.6))
      CALL EXIT
      END
```

```
SUBROUTINE CMULT(TRM,YTRM,P)
DIMENSION TRM(2),YTRM(2),P(1,2)
P(1,1)=TRM(1)*YTRM(1)-TRM(2)*YTRM(2)
P(1,2)=TRM(1)*YTRM(2)+TRM(2)*YTRM(1)
RETURN
END
```

This Fortran II program plots the real and imaginary parts of the current vector against the distance along the dipole. Plotting is done on a CALCOM plotter.

CUR(I,J,K)	-	A matrix of floating point numbers containing the array to be plotted.
I	-	row indexing.
J	-	column indexing.
K	-	real and imaginary indexing.
X	-	An array of floating point numbers which represents the distance along the dipole.

```
C     PROGRAM TO PLOT CURRENT DISTRIBUTION OF ARRAY.
      DIMENSION W(6),X(6)
      DATA X/0.0,0.05,0.10,0.15,0.20,0.25/
      DIMENSION CUR(25,4,2)
      READ(2,5)(((CUR(I,J,K),K=1,2),J=1,4),I=1,25)
 5    FORMAT(4E13.6)
      W(6)=0.0
      WRITE(5,6)
 6    FORMAT(1H1)
      DO 500 J=1,4
      CALL PSIZE(5.0,10.0)
      CALL PBOX
      CALL PAXES
      DO 500 K=1,2
      DO 501 N=1,5
      DO 515 M=1,5
      I=(N-1)*5+M
      W(M)=CUR(I,J,K)
 515  WRITE(5,1)CUR(I,J,K),W(M)
 1    FORMAT(2(2X,E13.6))
      CALL PLOS (0.0,X,0.25,-0.020,W,0.020,6)
 501  CONTINUE
      PAUSE
 500  CONTINUE
      CALL EXIT
      END
```

This Fortran II program computes the input to output power ratio of the array and the isolation between the center element and the circular array.

INPUT DATA:

A	-	radius of dipoles.
R	-	radius of the circular array.
X_5	-	value of the X-coordinate of the fifth element.
Y_5	-	value of the Y-coordinate of the fifth element.
E	-	percent error in displacement of the fifth element.
V(I,J,K)	-	voltage sequence.
CUR(I,J,K)	-	current distribution.

OUTPUT DATA:

All input specified above are written out.

PIN	-	input power to each dipole on the circle.
PO	-	output power to the fifth element.
PTOT	-	total input power.
PTR	-	power ratio.
PDB	-	isolation (in DB).

```
C      THIS PROGRMA COMPUTES THE INPUT AND OUTPUT POWER RATIO
C      FOR A FIVE ELEMENT ARRAY.
       REAL IO
       DIMENSION IO(2),CU(4,2)
       DIMENSION CUR(25,4,2),V(4,4,2),PIN(4,2),P(4,2)
       DIMENSICN TRM(2),YTRM(2),REP(4,4)
C      IMPEDANCE
       ZL=100.0
       DO 150 I=1,4
       DO 150 K=1,2
 150   CU(I,K)=0.0
C      READ IN PHASE
       READ(2,250)IFASE
 250   FORMAT(I1)
C      READ IN PARAMETERS FOR ARRAY
       READ(2,20)A,R,X5,Y5,E
  20   FORMAT(4F6.4,F5.2)
C      READ IN VOLTAGE SEQUENCE
       READ(2,230)(((V(I,J,K),K=1,2),I=1,4),J=1,4)
 230   FORMAT(8F4.1)
C      READ IN CURRENT DISTRIBUTION
       READ(2,101)(((CS(I,J,K),K=1,2),J=1,30),I=1,25)
 101   FORMAT(20A4)
       DO 102 I=1,25
       DO 102 J=1,30
       DO 102 K=1,2
 102   CS(I,J,K)=A4CVT(CS(I,J,K))
       DO 500 J=1,IFASE
       PTOT=0.0
       DO 300 I=1,4
       II=(I-1)*5+1
       DO 281 K=1,2
 281   CU(I,K)=CUR(II,J,K)
C      CONJUGATE CF I
       CUR(II,J,2)=-CUR(II,J,2)
       DO 325 K=1,2
       TRM(K)=CUR(II,J,K)
 325   YTRM(K)=V(I,J,K)
       CALL CMULT(TRM,YTRM,P)
       DO 327 N=1,2
 327   PIN(I,N)=P(1,N)
C      REAL POWER
       REP(I,J)=PIN(I,1)
       PTCT=PTOT+REP(I,J)
 300   CONTINUE
       DO  305 N=1,2
 305   IO(N)=CUR(21,J,N)
       ABSIO=IO(1)**2+IO(2)**2
```

```
      PO=ABSIO*ZL
      PTR=PO/PTOT
      PDB=4.343*ALOG(PTR)
      WRITE(5,50)
50    FORMAT(1H1)
      WRITE(5,355)J
355   FORMAT(5X,'PHASE=',I1,//)
      WRITE(5,25)A,R,X5,Y5,E
 25   FORMAT(5X,'A=',F8.4,/,5X,'R=',F8.4,/,5X,'X5=',F8.4,/,5X,'Y5=',F8.4
     C,/,5X,'ERROR=',F5.2,'  PERCENT',///)
      WRITE(5,400)((CU(I,K),K=1,2),I=1,4)
400   FORMAT(1X,'I1=',2F11.8,1X,'I2=',2F11.8,1X,'I3=',2F11.8,1X,'I4=',
     C2F11.8,///)
      WRITE(5,405)((V(I,J,K),K=1,2),I=1,4)
405   FORMAT(1X,'V1=',2F11.8,1X,'V2=',2F11.8,1X,'V3=',2F11.8,1X,'V4=',
     C2F11.8,///)
      WRITE(5,410)(REP(I,J),I=1,4)
410   FORMAT(1X,'P1=',F11.7,2X,'P2=',F11.7, 2X,'P3=',F11.7,2X,'P4=',F11.
     C7,//)
      WRITE(5,420)ZL,(IO(I),I=1,2),ABSIO
420   FORMAT(1X,'ZL=', F6.1,3X,'IO=',2E11.4,3X,'ABSIO=',E11.4,//)
      WRITE(5,425)PTOT,PO,PTR,PDB
425   FORMAT(1X,'PIN=',F11.7,3X,'PO=',F11.7,3X,'PO/PIN=',F11.7,3X,'PO/PI
     CN(DB)=',F11.7,///)
500   CONTINUE
      CALL EXIT
      END
```

This Fortran II program computes and plots the field pattern and phases in the phi plane.

INPUT DATA:

CUR(I,J,K)	-	A matrix of floating-point numbers containing the current at the respective nodes.
I	-	row indexing.
J	-	column indexing.
K	-	real or complex indexing.
X(L)	-	value of the X-coordinate of the L^{th} dipole.
Y(L)	-	value of the Y-coordinate of the L^{th} cipole.

OUTPUT DATA:

IFASE	-	phase sequence.
CUR(I,J,K)	-	as above.
X(L), Y(L)	-	as above.
FCOEF(NP,K)	-	complex field function.
EMAG(NP)	-	magnitude of FCOEF.
EDB	-	value of normalized EMAG in DB.
PSHFT(NP)	-	phase value to be plotted.

```
C       PROGRAM TO COMPUTE AND PLOT THE FIELD PATTERN AND PHASE
C       OF A FIVE ELEMENT CIRCULAR ARRAY IN THE PHI PLANE
        DIMENSION PHSFT(75)
        DIMENSION SI(75),EMAG(75),X1(75),Y1(75)
        DIMENSIONCUR(25,4,2),DCT(10),FASE(2),FCOEF(75,2),X(5),Y(5)
        DIMENSION TRM(2),YTRM(2),P(1,2)
        REAL M1
        PI=3.14159
        TWOPI=2.0*PI
        PI36=PI/36.0
        PIZ2=PI/2.0
        PI2=2.0*PI
        RAD=PI/180.0
        DELTA=0.0
        EH=.25
        DZ=EH/5.0
        THETA=PIZ2
        ASIN=SIN(THETA)
        ACOS=COS(THETA)
C       READ IN CURRENT AT RESPECTIVE NODES.
        READ(2,5)(((CUR(I,J,K),K=1,2),J=1,4),I=1,25)
 4      FORMAT(8(1X,E13.6))
 5      FORMAT(4E13.6)
C       READ IN CORDINATES OF DIPOLES.
        READ(2,10)(X(I),Y(I),I=1,5)
 10     FORMAT(2F6.4)
C       READ IN PHASE
        DO 999 IFASE=1,4
        EMAX=0.0
        WRITE(5,111)IFASE
 111    FORMAT(5X,'FASE='I2)
        WRITE(5,4)(((CUR(I,J,K),K=1,2),J=1,4),I=1,25)
        WRITE(5,3)(X(I),Y(I),I=1,5)
 3      FORMAT(2(1X,F8.4))
        WRITE(5,505)
 505    FORMAT(13X,'MAGNITUDE',15X,'PHASE')
        DO 2 I=1,73
        PHSFT(I)=0.0
        DO 2 J=1,2
 2      FCOFF(I,J)=0.0
         DO 9 NP=1,73
        PHI=(NP-1)*PI36
C       COMPONENTS OF UNIT VECTOR (A)
        AX=ASIN*COS(PHI)
        AY=ASIN*SIN(PHI)
        AZ=ACOS
        DO 24 I=1,5
        L=1
```

```
      DO 25 LL1=1,9
      L1=10-LL1
      M1=5-L1
C     VECTOR DOT PRODUCT
      DOT(L)=X(I)*AX+Y(I)*AY+M1*DZ*AZ
C     SOLID ANGLE
      SI(L)=TWOPI*DOT(L)
      FASE(1)=COS(SI(L))
      FASE(2)=SIN(SI(L))
      IF(LL1-5)6,14,7
 6    LL=-M1+(I-1)*5+1
      GO TO 11
 7    LL=M1+(I-1)*5+1
      GO TO 11
 14   LL=M1+(I-1)*5+1
 11   CONTINUE
      DO 30 N=1,2
      TRM(N)=CUR(LL,IFASE,N)
 30   YTRM(N)=FASE(N)
      CALL CMULT(TRM,YTRM,P)
      DO 35 N=1,2
 35   FCOEF(NP,N)=FCOEF(NP,N)+P(1,N)
 25   CONTINUE
 24   CONTINUE
      DO 36 N=1,2
 36   TRM(N)=FCOEF(NP,N)
      YTRM(1)=0.0
      YTRM(2)=60.0*PI*SIN(THETA)*DZ
      CALL CMULT(TRM,YTRM,P)
      DO 37 N=1,2
 37   FCOEF(NP,N)=P(1,N)
      X1(NP)=(NP-1)*PI36
      Z=FCOEF(NP,2)/FCOEF(NP,1)
      IF(FCOEF(NP,1).GT.0) GO TO 38
      IF(FCOEF(NP,2).GT.0) GO TO 39
      I=3
      ARG=ABS(ATAN(Z))
      GO TO 50
   39 I=2
      ARG=PIZ2-ABS(ATAN(Z))
      GO TO 50
 38   IF(FCOEF(NP,2).GT.0) GO TO 41
      I=4
      ARG=PIZ2-ABS(ATAN(Z))
      GO TO 50
   41 I=1
      ARG=ABS(ATAN(Z))
 50   PHSFT(NP)=ARG+(I-1)*PIZ2
      EMAG(NP)=SQRT(FCOEF(NP,1)**2+FCOEF(NP,2)**2)
```

```
      WRITE(5,510)EMAG(NP),PHSFT(NP)
510   FORMAT(9X,E13.6,11X,E13.6)
9     CONTINUE
      WRITE(2,500)(EMAG(NP),PHSFT(NP),NP=1,72)
500   FORMAT(5X,8F9.6)
999   CONTINUE
      CALL PSIZE(5.0,5.0)
      CALL PBOX
      CALL PAXES
      CALL PLOS (0.0,X1,TWOPI,0.0,PHSFT,10.0,72)
      DO 137 NP=1,73
      EDB=8.6860*ALOG(EMAG(NP)/EMAX)
137   WRITE(5,902)(FCOEF(NP,I),I=1,2),EMAG(NP),EDB
902   FORMAT(4(5X,E13.6))
      CALL PSIZE(5.0,5.0)
      CALL PBOX
      CALL PAXES
      DO 77 NP=1,7'
      X1(NP)=(NP-,,. '136
      Y1(NP)=EMAG(NP)/EMAX
77    CONTINUE
      CALL PLOTA(0.0,X1,TWOPI,0.0,Y1,1.0,72)
      PAUSE
      CALL EXIT
      END
```

This Fortran II program computes and plots the field pattern of the circular array in the Theta plane. (EMAG vs THETA.) Plotting is done on a CALCOM plotter.

INPUT DATA:

CUR(I,J,K)	-	A matrix of floating-point numbers containing the current at the respective nodes.
I	-	row indexing.
J	-	column indexing.
K	-	real or complex indexing.
X(L)	-	value of the X-coordinate of the L^{th} dipole.
Y(L)	-	value of the Y-coordinate of the L^{th} cipole.

OUTPUT DATA:

IFASE	-	phase sequence.
CUR(I,J,K)	-	as above.
X(L), Y(L)	-	as above.
FCOEF(NP,K)	-	complex field function.
EMAG(NP)	-	magnitude of FCOEF.
EDB	-	value of normalized EMAG in DB.

```
C     PROGRAM TO COMPUTE AND PLOT THE FIELD PATTERN
C     OF A FIVE ELEMENT CIRCULAR ARRAY IN THE THETA PLANE
      DIMENSION SI(75),EMAG(75),X1(75),Y1(75)
      DIMENSIONCUR(25,4,2),DOT(10),FASE(2),FCOEF(75,2),X(5),Y(5)
      DIMENSION TRM(2),YTRM(2),P(1,2)
      REAL M1
      PI=3.14159
      TWOPI=2.0*PI
      PI36=PI/36.0
      PIZ2=PI/2.0
      PI2=2.0*PI
      RAD=PI/180.0
      DELTA=0.0
      EH=.25
      DZ=EH/5.0
      PHI=0.0
      ACOS=COS(PHI)
      ASIN=SIN(PHI)
C     READ IN CURRENT AT RESPECTIVE NODES.
      READ(2,5)(((CUR(I,J,K),K=1,2),J=1,4),I=1,25)
 4    FORMAT(8(1X,E13.6))
 5    FORMAT(4E13.6)
C     READ IN CORDINATES OF DIPOLES.
      READ(2,10)(X(I),Y(I),I=1,5)
 10   FORMAT(2F6.4)
C     READ IN PHASE
      DO 999 IFASE=1,4
      EMAX=0.0
      WRITE(5,111)IFASE
 111  FORMAT(5X,'FASE='I2)
      WRITE(5,4)(((CUR(I,J,K),K=1,2),J=1,4),I=1,25)
      WRITE(5,3)(X(I),Y(I),I=1,5)
 3    FORMAT(2(1X,F8.4))
      WRITE(5,901)
 901  FORMAT(9X,'REAL',15X,'IMAG',15X,'MAG',15X,'DB')
      DO 2 I=1,37
      DO 2 J=1,2
 2    FCOEF(I,J)=0.0
      DO 9 NP=1,37
      THETA=(NP-1)*PI36
C     COMPONENTS OF UNIT VECTOR (A)
      AX=SIN(THETA)*ACOS
      AY=SIN(THETA)*ASIN
      AZ=COS(THETA)
      DO 24 I=1,5
      L=1
      DO 25 LL1=1,9
      L1=10-LL1
```

```
      M1=5-L1
C     VECTOR DOT PRODUCT
      DCT(L)=X(I)*AX+Y(I)*AY+M1*DZ*AZ
C     SOLID ANGLE
      SI(L)=TWOPI*DCT(L)
      FASE(1)=COS(SI(L))
      FASE(2)=SIN(SI(L))
      IF(LL1-5)6,14,7
 6    LL=-M1+(I-1)*5+1
      GO TO 11
 7    LL=M1+(I-1)*5+1
      GO TO 11
 14   LL=M1+(I-1)*5+1
 11   CONTINUE
      DO 30 N=1,2
      TRM(N)=CUR(LL,IFASE,N)
 30   YTRM(N)=FASE(N)
      CALL CMULT(TRM,YTRM,P)
      DO 35 N=1,2
 35   FCOFF(NP,N)=FCOEF(NP,N)+P(1,N)
 25   CONTINUE
 24   CONTINUE
      DO 36 N=1,2
 36   TRM(N)=FCOEF(NP,N)
      YTRM(1)=0.0
      YTRM(2)=60.0*PI*SIN(THETA)*DZ
      CALL CMULT(TRM,YTRM,P)
      DO 37 N=1,2
 37   FCOEF(NP,N)=P(1,N)
      EMAG(NP)=SQRT(FCOEF(NP,1)**2+FCOEF(NP,2)**2)
      IF(EMAX-EMAG(NP))138,9,9
 138  EMAX=EMAG(NP)
 9    CONTINUE
      DO 137 NP=1,37
      FDB=8.6860*ALOG(EMAG(NP)/EMAX)
 137  WRITE(5,902)(FCOEF(NP,I),I=1,2),EMAG(NP),EDB
 902  FORMAT(4(5X,E13.6))
      CALL PSIZE(5.0,5.0)
      CALL PBOX
      CALL PAXES
      DO 77 NP=1,37
      X1(NP)=(NP-1)*PI36
      Y1(NP)=EMAG(NP)/EMAX
 77   CONTINUE
      CALL PLOTA(0.0,X1,PI,0.0,Y1,1.0,36)
      PAUSE
 999  CONTINUE
      CALL EXIT
      END
```

APPENDIX D

THEORETICAL DATA

Data that were not included in the text are appended here for reference. They include plots of current distributions, tables of short-circuited admittances, and radiation patterns.

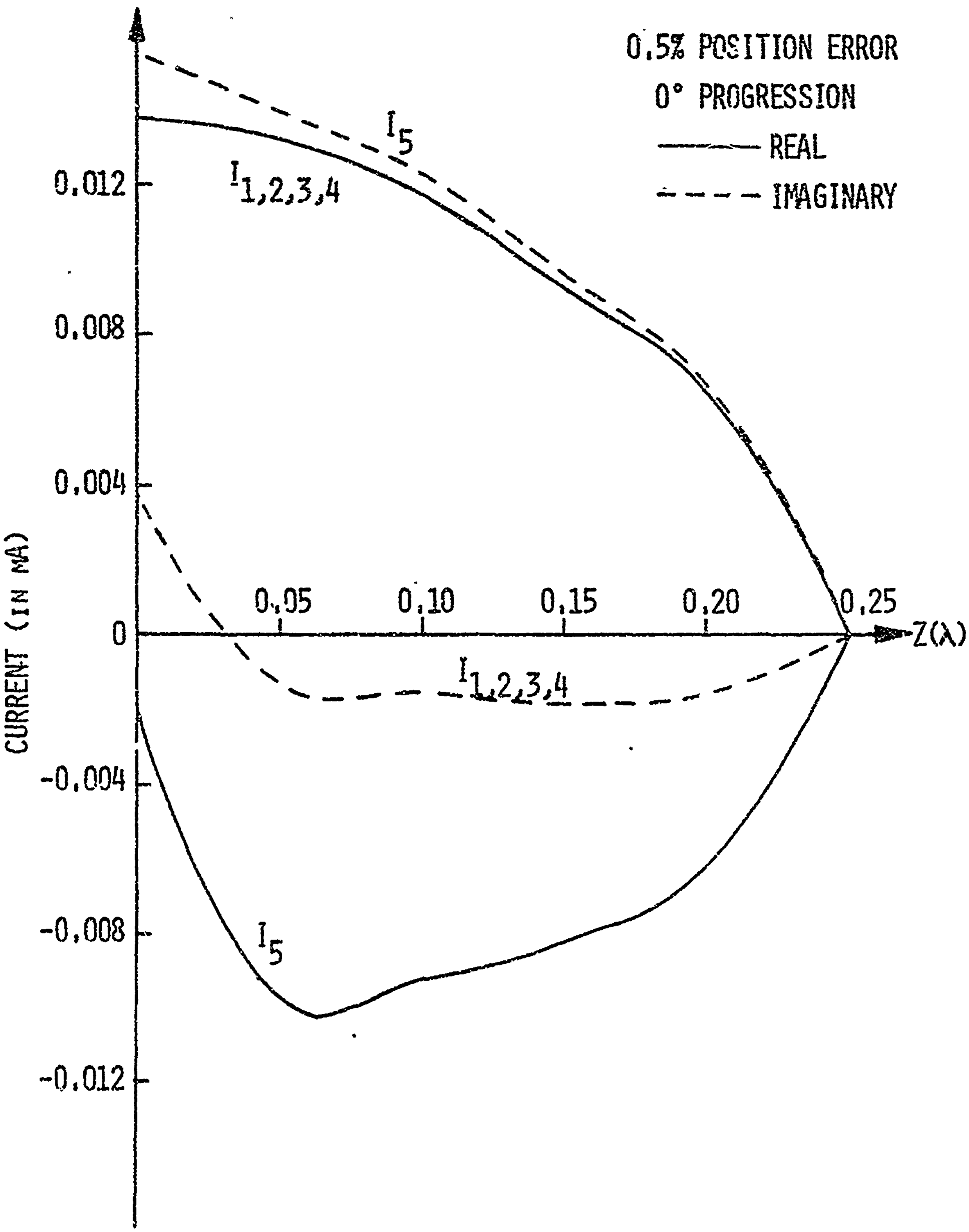


Figure D-1: Current Distribution for the C° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025λ, and Position Error is 0.5%.

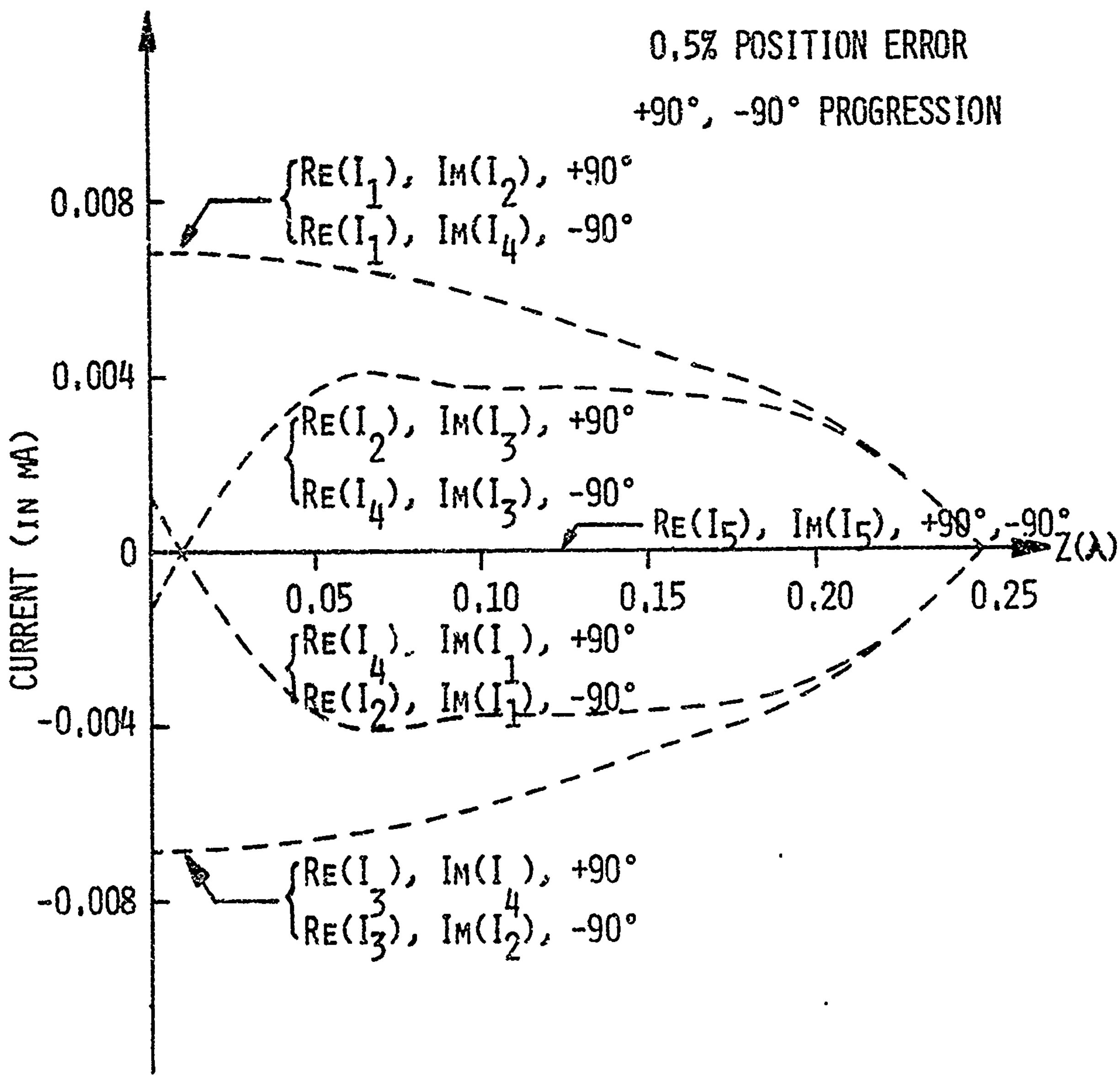


Figure D-2: Current Distribution for the +90° and -90° Phase Progressions. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025λ, and Position Error is 0.5%.

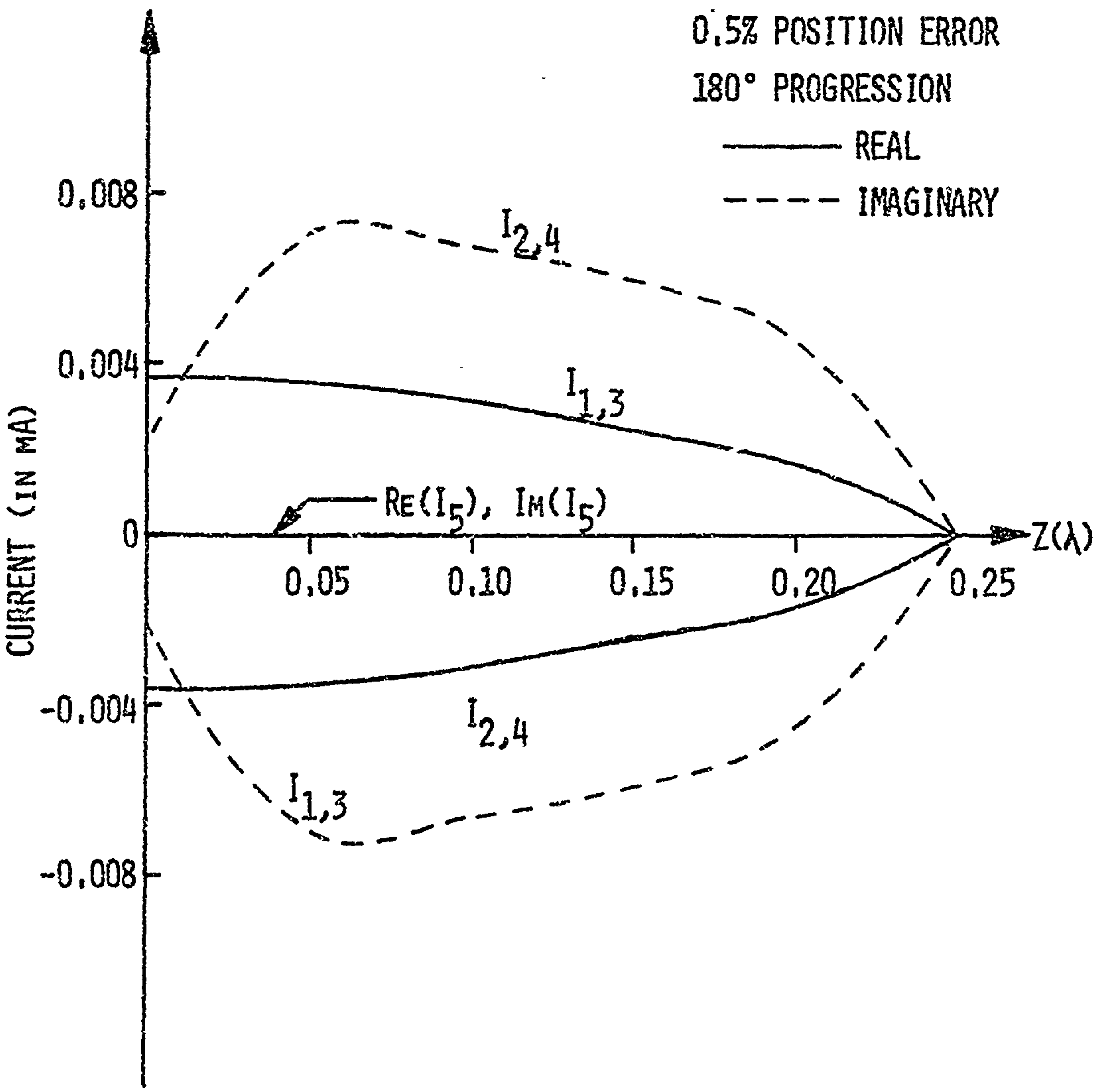


Figure D-3: Current Distribution for the 180° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 0.5%.

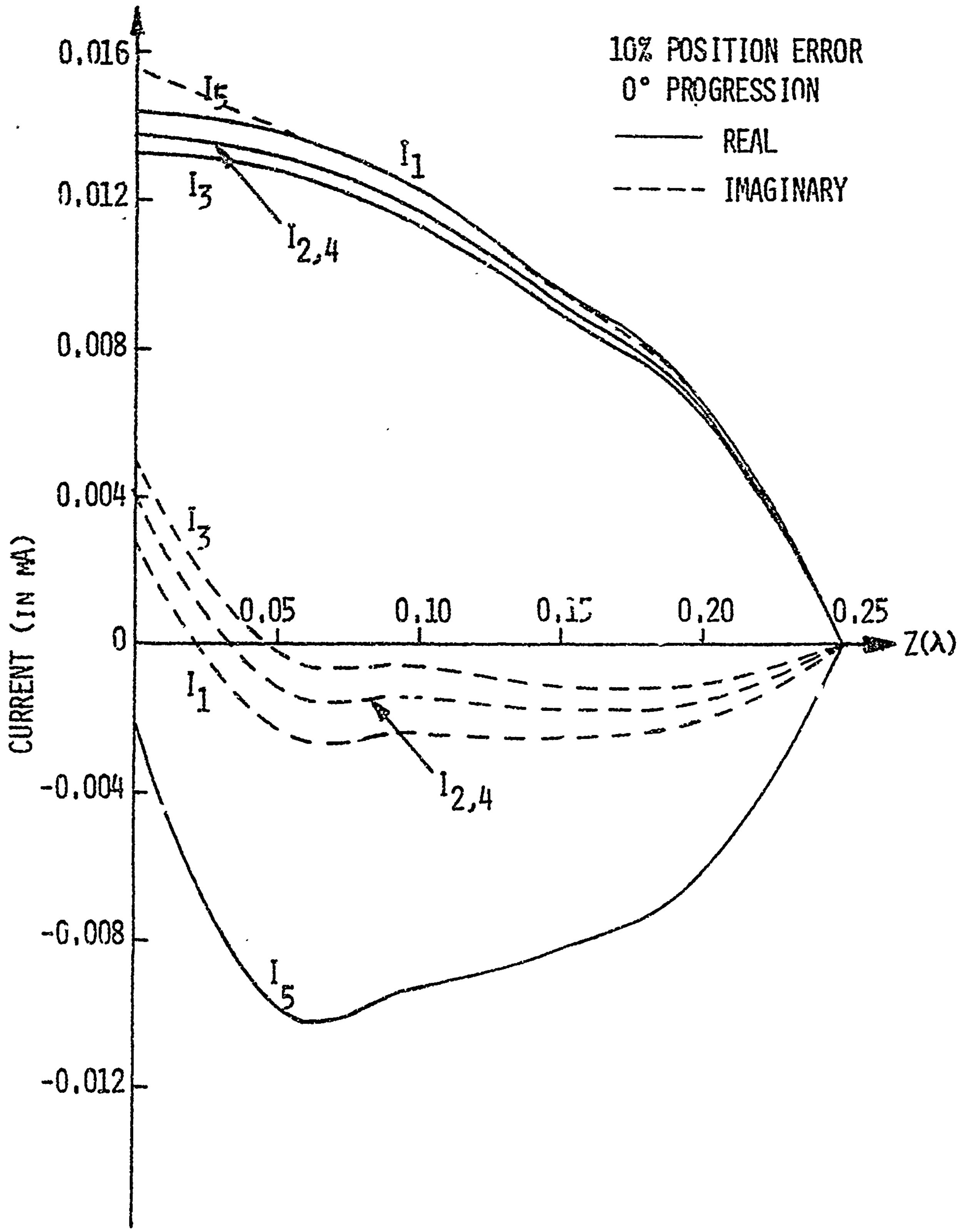


Figure D-4: Curren. Distribution for the 0° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 10%.

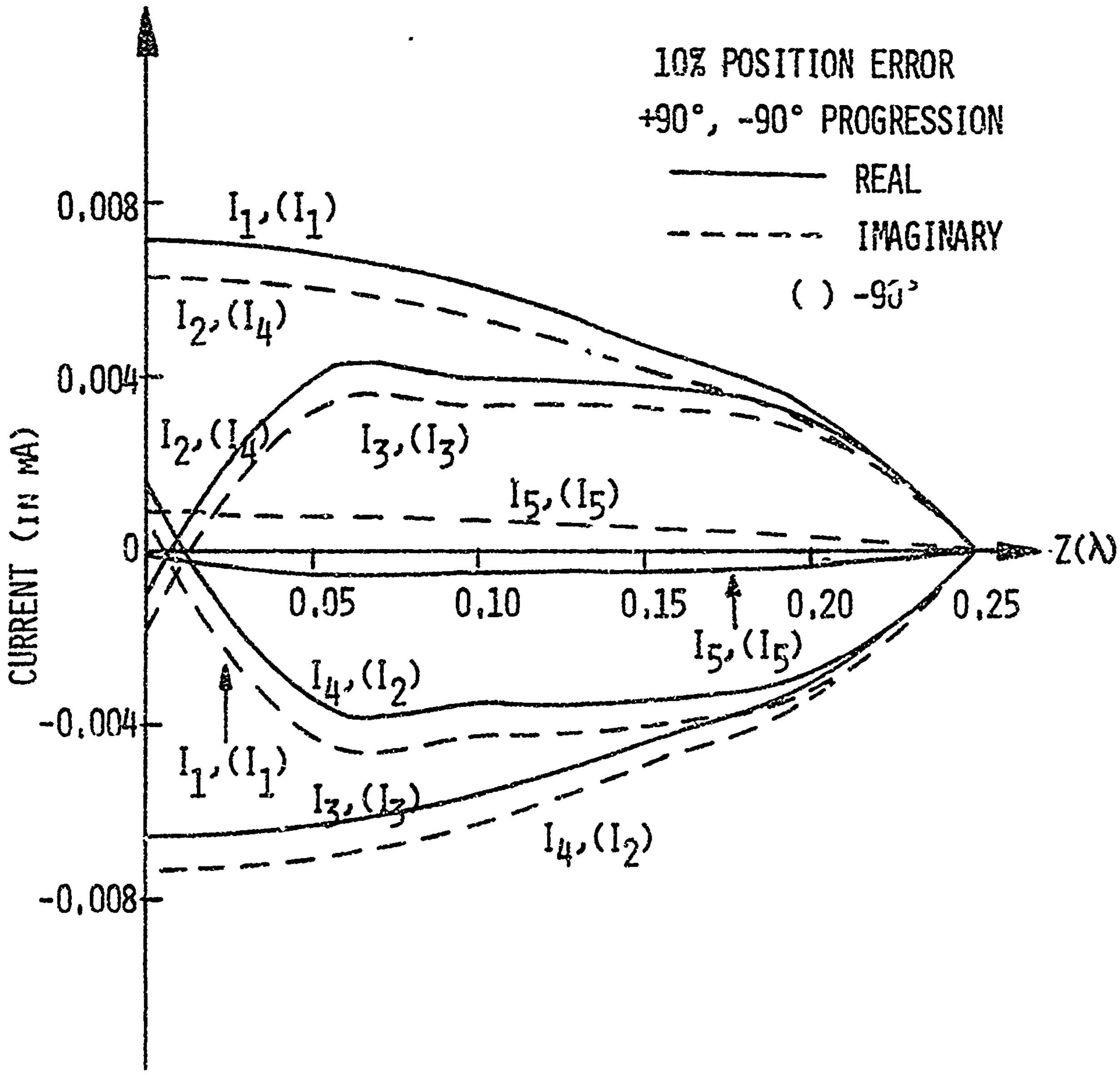


Figure D-5: Current Distribution for the +90° and -90° Phase Progressions. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 10%.

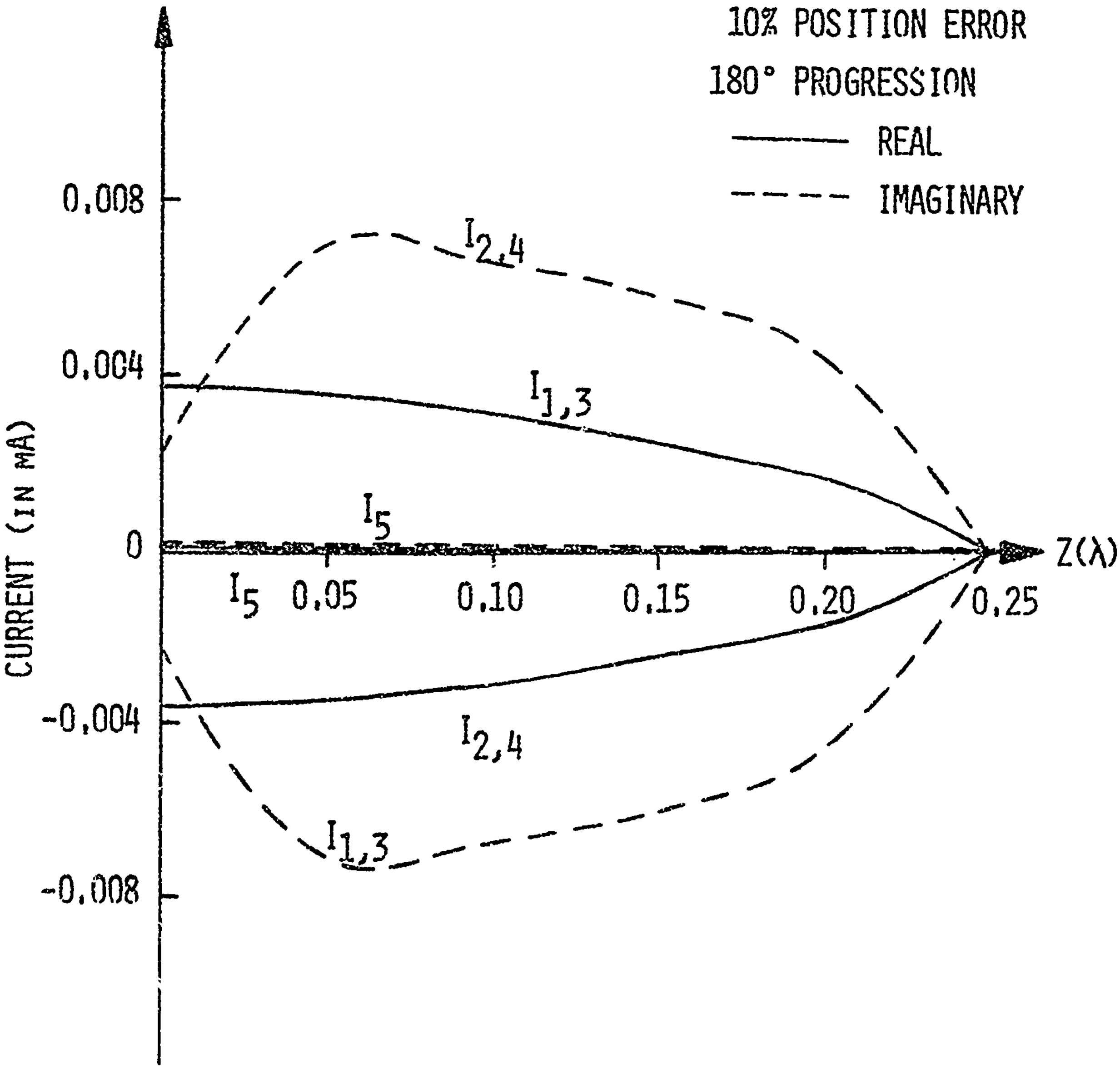


Figure D-6: Current Distribution for the 180° Phase Progression. Radius (R) of the Array is 0.3 λ, Radius of the Dipole (A) is 0.025 λ, and Position Error is 10%.

$A = 0.0250$
$R = 0.5000$
$X_5 = 0.0025$
$Y_5 = 0.0000$

ERROR = 0.5%

Y_{11} = 0.12007E-01+j0.20060E-02	Y_{12} = 0.33829E-02-j0.35216E-03
Y_{13} = 0.55746E-03+j0.91187E-04	Y_{14} = 0.33829E-02-j0.35216E-03
Y_{15} = 0.68221E-02+j0.37664E-02	Y_{21} = 0.33829E-02-j0.35217E-03
Y_{22} = 0.12103E-01+j0.19619E-02	Y_{23} = 0.34782E-02-j0.39713E-03
Y_{24} = 0.55796E-03+j0.90690E-04	Y_{25} = 0.69145E-02+j0.37184E-02
Y_{31} = 0.55746E-03+j0.91186E-04	Y_{32} = 0.34782E-02-j0.39713E-03
Y_{33} = 0.12198E-01+j0.19161E-02	Y_{34} = 0.34782E-02-j0.39713E-03
Y_{35} = 0.70062E-02+j0.36699E-02	Y_{41} = 0.33829E-02-j0.35217E-03
Y_{42} = 0.55796E-03+j0.90690E-04	Y_{43} = 0.34782E-02-j0.39713E-03
Y_{44} = 0.12103E-01+j0.19619E-02	Y_{45} = 0.69145E-02+j0.37184E-02
Y_{51} = 0.68221E-02+j0.37664E-02	Y_{52} = 0.69145E-02+j0.37184E-02
Y_{53} = 0.70062E-02+j0.36699E-02	Y_{54} = 0.69145E-02+j0.37184E-02
Y_{55} = 0.13993E-01+j0.11951E-01	

Z_{11} = 0.11073E+03-j0.11507E+02	Z_{12} =-0.15186E+01+j0.28999E+02
Z_{13} = 0.26035E+02+j0.19670E+01	Z_{14} =-0.15186E+01+j0.28999E+02
Z_{15} =-0.60029E+02-j0.85910E+01	Z_{21} =-0.15186E+01+j0.28999E+02
Z_{22} = 0.11043E+03-j0.11714E+02	Z_{23} =-0.18166E+01+j0.28796E+02
Z_{24} = 0.26033E+02+j0.19663E+01	Z_{25} =-0.59851E+02-j0.80004E+01
Z_{31} = 0.26035E+02+j0.19670E+01	Z_{32} =-0.18167E+01+j0.28796E+02
Z_{33} = 0.11014E+03-j0.11912E+02	Z_{34} =-0.18167E+01+j0.28796E+02
Z_{35} =-0.59670E+02-j0.74185E+01	Z_{41} =-0.15186E+01+j0.28999E+02
Z_{42} = 0.26033E+02+j0.19663E+01	Z_{43} =-0.18166E+01+j0.28796E+02
Z_{44} = 0.11043E+03-j0.11714E+02	Z_{45} =-0.59851E+02-j0.80004E+01
Z_{51} =-0.60029E+02-j0.85910E+01	Z_{52} =-0.59851E+02-j0.80004E+01
Z_{53} =-0.59670E+02-j0.74185E+01	Z_{54} =-0.59851E+02-j0.80004E+01
Z_{55} = 0.14402E+03-j0.43584E+02	

Table D1: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.5000
X_5= 0.0100
Y_5= 0.0000

ERROR = 2.0%

Y_{11} = 0.11725E-01+j0.21175E-02	Y_{12} = 0.32439E-02-j0.29677E-03
Y_{13} = 0.55663E-03+j0.87410E-04	Y_{14} = 0.32439E-02-j0.29677E-03
Y_{15} = 0.65553E-02+j0.38881E-02	Y_{21} = 0.32439E-02-j0.29677E-03
Y_{22} = 0.12109E-01+j0.19507E-02	Y_{23} = 0.36245E-02-j0.47782E-03
Y_{24} = 0.56458E-03+j0.79495E-04	Y_{25} = 0.69292E-02+j0.36966E-02
Y_{31} = 0.55663E-03+j0.87410E-04	Y_{32} = 0.36245E-02-j0.47782E-03
Y_{33} = 0.12485E-01+j0.17557E-02	Y_{34} = 0.36245E-02-j0.47782E-03
Y_{35} = 0.72905E-02+j0.34996E-02	Y_{41} = 0.32439E-02-j0.29677E-03
Y_{42} = 0.56458E-03+j0.79495E-04	Y_{43} = 0.36245E-02-j0.47782E-03
Y_{44} = 0.12109E-01+j0.19507E-02	Y_{45} = 0.69292E-02+j0.36966E-02
Y_{51} = 0.65553E-02+j0.38881E-02	Y_{52} = 0.69292E-02+j0.36966E-02
Y_{53} = 0.72905E-02+j0.34996E-02	Y_{54} = 0.69292E-02+j0.36966E-02
Y_{55} = 0.14025E-01+j0.11909E-01	
Z_{11} = 0.11162E+03-j0.10831E+02	Z_{12} =-0.10830E+01+j0.29324E+02
Z_{13} = 0.26035E+02+j0.19800E+01	Z_{14} =-0.10830E+01+j0.29324E+02
Z_{15} =-0.60528E+02-j0.10416E+02	Z_{21} =-0.10830E+01+j0.29324E+02
Z_{22} = 0.11042E+03-j0.11712E+02	Z_{23} =-0.22741E+01+j0.28513E+02
Z_{24} = 0.26015E+02+j0.19679E+01	Z_{25} =-0.59835E+02-j0.80002E+01
Z_{31} = 0.26035E+02+j0.19800E+01	Z_{32} =-0.22741E+01+j0.28513E+02
Z_{33} = 0.10924E+03-j0.12454E+02	Z_{34} =-0.22741E+01+j0.28513E+02
Z_{35} =-0.59095E+02-j0.57250E+01	Z_{41} =-0.10830E+01+j0.29324E+02
Z_{42} = 0.26015E+02+j0.19680E+01	Z_{43} =-0.22741E+01+j0.28513E+02
Z_{44} = 0.11042E+03-j0.11712E+02	Z_{45} =-0.59835E+02-j0.80002E+01
Z_{51} =-0.60528E+02-j0.10416E+02	Z_{52} =-0.59835E+02-j0.80002E+01
Z_{53} =-0.59095E+02-j0.57249E+01	Z_{54} =-0.59835E+02-j0.80002E+01
Z_{55} = 0.14400E+03-j0.43456E+02	

Table D2: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.5000
X_5 = 0.0500
Y_5 = 0.0000

ERROR = 10.0%

Y_{11} = 0.10336E-01+j0.23362E-02	Y_{12} = 0.25917E-02-j0.19167E-03
Y_{13} = 0.52673E-03-j0.62409E-05	Y_{14} = 0.25917E-02-j0.19167E-03
Y_{15} = 0.53309E-02+j0.41789E-02	Y_{21} = 0.25917E-02-j0.19167E-03
Y_{22} = 0.12253E-01+j0.16524E-02	Y_{23} = 0.44076E-02-j0.12402E-02
Y_{24} = 0.70792E-03-j0.21881E-03	Y_{25} = 0.72550E-02+j0.31139E-02
Y_{31} = 0.52673E-03-j0.62414E-05	Y_{32} = 0.44076E-02-j0.12402E-02
Y_{33} = 0.13891E-01+j0.29074E-03	Y_{34} = 0.44076E-02-j0.12402E-02
Y_{35} = 0.88506E-02+j0.19367E-02	Y_{41} = 0.25917E-02-j0.19167E-03
Y_{42} = 0.70792E-03-j0.21881E-03	Y_{43} = 0.44076E-02-j0.12402E-02
Y_{44} = 0.12253E-01+j0.16524E-02	Y_{45} = 0.72550E-02+j0.31139E-02
Y_{51} = 0.53309E-02+j0.41789E-02	Y_{52} = 0.72550E-02+j0.31139E-02
Y_{53} = 0.88506E-02+j0.19367E-02	Y_{54} = 0.72550E-02+j0.31139E-02
Y_{55} = 0.14756E-01+j0.10760E-01	

Z_{11} = 0.11583E+03-j0.57888E+01	Z_{12} = 0.94846E+00+j0.31453E+02
Z_{13} = 0.26046E+02+j0.23150E+01	Z_{14} = 0.94846E+00+j0.31453E+02
Z_{15} =-0.62163E+02-j0.21522E+02	Z_{21} = 0.94846E+00+j0.31453E+02
Z_{22} = 0.10995E+03-j0.11666E+02	Z_{23} =-0.48784E+01+j0.27440E+02
Z_{24} = 0.25551E+02+j0.20143E+01	Z_{25} =-0.59397E+02-j0.79888E+01
Z_{31} = 0.26046E+02+j0.23150E+01	Z_{32} =-0.48784E+01+j0.27440E+02
Z_{33} = 0.10450E+03-j0.14115E+02	Z_{34} =-0.48784E+01+j0.27440E+02
Z_{35} =-0.55428E+02+j0.20329E+01	Z_{41} =-0.94846E+00+j0.31453E+02
Z_{42} = 0.25551E+02+j0.20143E+01	Z_{43} =-0.48784E+01+j0.27440E+02
Z_{44} = 0.10995E+03-j0.11666E+02	Z_{45} =-0.59397E+02-j0.79888E+01
Z_{51} =-0.62163E+02-j0.21522E+02	Z_{52} =-0.59397E+02-j0.79888E+01
Z_{53} =-0.55428E+02+j0.20330E+01	Z_{54} =-0.59397E+02-j0.79888E+01
Z_{55} = 0.14338E+03-j0.40196E+02	

Table D3: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.5000
X_5= 0.0018
Y_5= 0.0018

ERROR = 0.5%

Y_{11} = 0.12034E-01+j0.19938E-02	Y_{12} = 0.33620E-02-j0.34266E-03
Y_{13} = 0.55772E-03+j0.90923E-04	Y_{14} = 0.34305E-02-j0.37432E-03
Y_{15} = 0.68481E-02+j0.37530E-02	Y_{21} = 0.33620E-02-j0.34265E-03
Y_{22} = 0.12034E-01+j0.19933E-02	Y_{23} = 0.34305E-02-j0.37432E-03
Y_{24} = 0.55772E-03+j0.90923E-04	Y_{25} = 0.68481E-02+j0.37530E-02
Y_{31} = 0.55772E-03+j0.90924E-04	Y_{32} = 0.34305E-02-j0.37432E-03
Y_{33} = 0.12171E-01+j0.19290E-02	Y_{34} = 0.34992E-02-j0.40741E-03
Y_{35} = 0.69806E-02+j0.36835E-02	Y_{41} = 0.34305E-02-j0.37432E-03
Y_{42} = 0.55772E-03+j0.90924E-04	Y_{43} = 0.34992E-02-j0.40741E-03
Y_{44} = 0.12171E-01+j0.19290E-02	Y_{45} = 0.69806E-02+j0.36835E-02
Y_{51} = 0.68481E-02+j0.37530E-02	Y_{52} = 0.68481E-02+j0.37530E-02
Y_{53} = 0.69806E-02+j0.36835E-02	Y_{54} = 0.69506E-02+j0.36835E-02
Y_{55} = 0.13993E-01+j0.11951E-01	

Z_{11} = 0.11065E+03-j0.11566E+02	Z_{12} =-0.14533E+01+j0.29045E+02
Z_{13} = 0.26034E+02+j0.19666E+01	Z_{14} =-0.16675E+01+j0.28897E+02
Z_{15} =-0.59979E+02-j0.84248E+01	Z_{21} =-0.14532E+01+j0.29044E+02
Z_{22} = 0.11065E+03-j0.11566E+02	Z_{23} =-0.16675E+01+j0.28897E+02
Z_{24} = 0.26034E+02+j0.19666E+01	Z_{25} =-0.59979E+02-j0.84248E+01
Z_{31} = 0.26034E+02+j0.19666E+01	Z_{32} =-0.16675E+01+j0.28897E+02
Z_{33} = 0.11022E+03-j0.11857E+02	Z_{34} =-0.18823E+01+j0.28753E+02
Z_{35} =-0.59721E+02-j0.75805E+01	Z_{41} =-0.16675E+01+j0.28897E+02
Z_{42} = 0.26034E+02+j0.19666E+01	Z_{43} =-0.18823E+01+j0.28753E+02
Z_{44} = 0.11022E+03-j0.11857E+02	Z_{45} =-0.59721E+02-j0.75805E+01
Z_{51} =-0.59979E+02-j0.84247E+01	Z_{52} =-0.59979E+02-j0.84248E+01
Z_{53} =-0.59721E+02-j0.75804E+01	Z_{54} =-0.59721E+02-j0.75805E+01
Z_{55} = 0.14402E+03-j0.43584E+02	

Table D4: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.5000
X_5= 0.0071
Y_5= 0.0071

ERROR = 2%

Y_{11} = 0.11837E-01+j0.20721E-02	Y_{12} = 0.31649E-02-j0.26437E-03
Y_{13} = 0.56063E-03+j0.83385E-04	Y_{14} = 0.34334E-02-j0.38186E-03
Y_{15} = 0.66652E-02+j0.38331E-02	Y_{21} = 0.31649E-02-j0.26436E-03
Y_{22} = 0.11837E-01+j0.20721E-02	Y_{23} = 0.34334E-02-j0.38186E-03
Y_{24} = 0.56063E-03+j0.83383E-04	Y_{25} = 0.66652E-02+j0.38331E-02
Y_{31} = 0.56062E-03+j0.83385E-04	Y_{32} = 0.34334E-02-j0.38186E-03
Y_{33} = 0.12377E-01+j0.18149E-02	Y_{34} = 0.37052E-02-j0.52156E-03
Y_{35} = 0.71871E-02+j0.35570E-02	Y_{41} = 0.34334E-02-j0.38186E-03
Y_{42} = 0.56062E-03+j0.83382E-04	Y_{43} = 0.37052E-02-j0.52156E-03
Y_{44} = 0.12377E-01+j0.18149E-02	Y_{45} = 0.71871E-02+j0.35570E-02
Y_{51} = 0.66652E-02+j0.38331E-02	Y_{52} = 0.66652E-02+j0.38331E-02
Y_{53} = 0.71871E-02+j0.35570E-02	Y_{54} = 0.71871E-02+j0.35570E-02
Y_{55} = 0.14025E-01+j0.11908E-01	

Z_{11} = 0.11127E+03-j0.11101E+02	Z_{12} =-0.83570E+00+j0.29509E+02
Z_{13} = 0.26025E+02+j0.19740E+01	Z_{14} =-0.16761E+01+j0.28904E+02
Z_{15} =-0.60332E+02-j0.97010E+01	Z_{21} =-0.83569E+00+j0.29509E+02
Z_{22} = 0.11127E+03-j0.11101E+02	Z_{23} =-0.16761E+01+j0.28904E+02
Z_{24} = 0.26025E+02+j0.19740E+01	Z_{25} =-0.60332E+02-j0.97010E+01
Z_{31} = 0.26025E+02+j0.19740E+01	Z_{32} =-0.16761E+01+j0.28904E+02
Z_{33} = 0.10958E+03-j0.12252E+02	Z_{34} =-0.25266E+01+j0.28358E+02
Z_{35} =-0.59313E+02-j0.63704E+01	Z_{41} =-0.16761E+01+j0.28904E+02
Z_{42} = 0.26025E+02+j0.19740E+01	Z_{43} =-0.25266E+01+j0.28358E+02
Z_{44} = 0.10958E+03-j0.12252E+02	Z_{45} =-0.59313E+02-j0.63705E+01
Z_{51} =-0.60332E+02-j0.97010E+01	Z_{52} =-0.60332E+02-j0.97010E+01
Z_{53} =-0.59313E+02-j0.63705E+01	Z_{54} =-0.59313E+02-j0.63704E+01
Z_{55} = 0.14400E+03-j0.43455E+02	

Table D5: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.5000
X_5= 0.0354
Y_5= 0.0354

ERROR = 10%

Y_{11} = 0.10912E-01+j0.22251E-02	Y_{12} = 0.22402E-02-j0.11138E-03
Y_{13} = 0.61713E-03-j0.11477E-03	Y_{14} = 0.34899E-02-j0.58002E-03
Y_{15} = 0.59290E-02+j0.38859E-02	Y_{21} = 0.22402E-02-j0.11138E-03
Y_{22} = 0.10912E-01+j0.22251E-02	Y_{23} = 0.34899E-02-j0.58002E-03
Y_{24} = 0.61713E-03-j0.11477E-03	Y_{25} = 0.59290E-02+j0.38859E-02
Y_{31} = 0.61713E-03-j0.11477E-03	Y_{32} = 0.34899E-02-j0.58002E-03
Y_{33} = 0.13443E-01+j0.73330E-03	Y_{34} = 0.47710E-02-j0.16032E-02
Y_{35} = 0.84057E-02+j0.22736E-02	Y_{41} = 0.34899E-02-j0.58002E-03
Y_{42} = 0.61713E-03-j0.11477E-03	Y_{43} = 0.47710E-02-j0.16032E-02
Y_{44} = 0.13443E-01+j0.73330E-03	Y_{45} = 0.84057E-02+j0.22736E-02
Y_{51} = 0.59290E-02+j0.38859E-02	Y_{52} = 0.59290E-02+j0.38859E-02
Y_{53} = 0.84057E-02+j0.22736E-02	Y_{54} = 0.84057E-02+j0.22736E-02
Y_{55} = 0.14742E-01+j0.10747E-01	

Z_{11} = 0.11418E+03-j0.79510E+01	Z_{12} = 0.20759E+01+j0.32659E+02
Z_{13} = 0.25801E+02+j0.21559E+01	Z_{14} =-0.19004E+01+j0.29086E+02
Z_{15} =-0.61582E+02-j0.17187E+02	Z_{21} = 0.20759E+01+j0.32659E+02
Z_{22} = 0.11418E+03-j0.79511E+01	Z_{23} =-0.19003E+01+j0.29086E+02
Z_{24} = 0.25801E+02+j0.21558E+01	Z_{25} =-0.61582E+02-j0.17187E+02
Z_{31} = 0.25801E+02+j0.21558E+01	Z_{32} =-0.19003E+01+j0.29086E+02
Z_{33} = 0.10597E+03-j0.13671E+02	Z_{34} =-0.61323E+01+j0.26939E+02
Z_{35} =-0.56646E+02-j0.53209E+00	Z_{41} =-0.19004E+01+j0.29086E+02
Z_{42} = 0.25801E+02+j0.21558E+01	Z_{43} =-0.61322E+01+j0.26939E+02
Z_{44} = 0.10597E+03-j0.13671E+02	Z_{45} =-0.56646E+02-j0.53207E+00
Z_{51} =-0.61582E+02-j0.17187E+02	Z_{52} =-0.61582E+02-j0.17187E+02
Z_{53} =-0.56646E+02-j0.53204E+00	Z_{54} =-0.56646E+02-j0.53209E+00
Z_{55} = 0.14344E+03-j0.40199E+02	

Table D6: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.3000
X_5 = 0.0011
Y_5 = 0.0011

ERROR = 0.5%

Y_{11} = 0.61673E-02+j0.10064E-02	Y_{12} = 0.90189E-03+j0.12848E-02
Y_{13} =-0.67796E-03-j0.42668E-03	Y_{14} = 0.90513E-03+j0.13059E-02
Y_{15} = 0.74428E-04+j0.41473E-02	Y_{21} = 0.90189E-03+j0.12848E-02
Y_{22} = 0.61673E-02+j0.10064E-02	Y_{23} = 0.90513E-03+j0.13059E-02
Y_{24} =-0.67796E-03-j0.42668E-03	Y_{25} = 0.74426E-04+j0.41473E-02
Y_{31} =-0.67796E-03-j0.42668E-03	Y_{32} = 0.90513E-03+j0.13059E-02
Y_{33} = 0.61738E-02+j0.10484E-02	Y_{34} = 0.90834E-03+j0.13268E-02
Y_{35} = 0.72614E-04+j0.41131E-02	Y_{41} = 0.90513E-03+j0.13059E-02
Y_{42} =-0.67796E-03-j0.42668E-03	Y_{43} = 0.90834E-03+j0.13268E-02
Y_{44} = 0.61738E-02+j0.10484E-02	Y_{45} = 0.72613E-04+j0.41131E-02
Y_{51} = 0.74430E-04+j0.41473E-02	Y_{52} = 0.74428E-04+j0.41473E-02
Y_{53} = 0.72615E-04+j0.41131E-02	Y_{54} = 0.72614E-04+j0.41131E-02
Y_{55} = 0.45548E-03-j0.15182E-02	

Z_{11} = 0.12418E+03+j0.91450E+01	Z_{12} =-0.50275E+02-j0.33139E+02
Z_{13} =-0.15343E+02+j0.38776E+02	Z_{14} =-0.50077E+02-j0.33175E+02
Z_{15} = 0.77707E+01-j0.53196E+02	Z_{21} =-0.50275E+02-j0.33139E+02
Z_{22} = 0.12418E+03+j0.91451E+01	Z_{23} =-0.50077E+02-j0.33175E+02
Z_{24} =-0.15343E+02+j0.38776E+02	Z_{25} = 0.77706E+01-j0.53196E+02
Z_{31} =-0.15343E+02+j0.38776E+02	Z_{32} =-0.50077E+02-j0.33175E+02
Z_{33} = 0.12457E+03+j0.90700E+01	Z_{34} =-0.49881E+02-j0.33214E+02
Z_{35} = 0.69860E+01-j0.52572E+02	Z_{41} =-0.50077E+02-j0.33175E+02
Z_{42} =-0.15343E+02+j0.38776E+02	Z_{43} =-0.49881E+02-j0.33214E+02
Z_{44} = 0.12457E+03+j0.90700E+01	Z_{45} = 0.69859E+01-j0.52572E+02
Z_{51} = 0.77707E+01-j0.53196E+02	Z_{52} = 0.77705E+01-j0.53196E+02
Z_{53} = 0.69860E+01-j0.52572E+02	Z_{54} = 0.69860E+01-j0.52572E+02
Z_{55} = 0.86780E+02+j0.55719E+02	

Table D7: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.3000
X_5= 0.0042
Y_5= 0.0042

ERROR = 2%

Y_{11} = 0.61581E-02+j0.94689E-03	Y_{12} = 0.89263E-03+j0.12252E-02
Y_{13} =-0.67789E-03-j0.42592E-03	Y_{14} = 0.90520E-03+j0.13066E-02
Y_{15} = 0.77093E-04+j0.41961E-02	Y_{21} = 0.89263E-03+j0.12252E-02
Y_{22} = 0.61581E-02+j0.94689E-03	Y_{23} = 0.90520E-03+j0.13066E-02
Y_{24} =-0.67789E-03-j0.42592E-03	Y_{25} = 0.77091E-04+j0.41961E-02
Y_{31} =-0.67789E-03-j0.42592E-03	Y_{32} = 0.90520E-03+j0.13066E-02
Y_{33} = 0.61827E-02+j0.11071E-02	Y_{34} = 0.91728E-03+j0.13855E-02
Y_{35} = 0.70126E-04+j0.40657E-02	Y_{41} = 0.90520E-03+j0.13066E-02
Y_{42} =-0.67789E-03-j0.42592E-03	Y_{43} = 0.91728E-03+j0.13855E-02
Y_{44} = 0.61827E-02+j0.11072E-02	Y_{45} = 0.70126E-04+j0.40657E-02
Y_{51} = 0.77091E-04+j0.41961E-02	Y_{52} = 0.77091E-04+j0.41961E-02
Y_{53} = 0.70125E-04+j0.40657E-02	Y_{54} = 0.70125E-04+j0.40657E-02
Y_{55} = 0.45555E-03-j0.15210E-02	

Z_{11} = 0.12362E+03+j0.92371E+01	Z_{12} =-0.50835E+02-j0.33047E+02
Z_{13} =-0.15327E+02+j0.38776E+02	Z_{14} =-0.50061E+02-j0.33175E+02
Z_{15} = 0.88689E+01-j0.54045E+02	Z_{21} =-0.50835E+02-j0.33047E+02
Z_{22} = 0.12362E+03+j0.92371E+01	Z_{23} =-0.50061E+02-j0.33175E+02
Z_{24} =-0.15327E+02+j0.38776E+02	Z_{25} = 0.88688F+01-j0.54044E+02
Z_{31} =-0.15327E+02+j0.38776E+02	Z_{32} =-0.50061E+02-j0.33175E+02
Z_{33} = 0.12512E+03+j0.89501E+01	Z_{34} =-0.49332E+02-j0.33333E+02
Z_{35} = 0.58749E+01-j0.51665E+02	Z_{41} =-0.50061E+02-j0.33175E+02
Z_{42} =-0.15327E+02+j0.38776E+02	Z_{43} =-0.49332E+02-j0.33333E+02
Z_{44} = 0.12512E+03+j0.89502E+01	Z_{45} = 0.58748E+01-j0.51665E+02
Z_{51} = 0.88689E+01-j0.54045E+02	Z_{52} = 0.88688E+01-j0.54044E+02
Z_{53} = 0.58749E+01-j0.51665E+02	Z_{54} = 0.58748E+01-j0.51665E+02
Z_{55} = 0.86762E+02+j0.55634E+02	

Table D8: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0250
R = 0.3000
X_5 = 0.0212
Y_5 = 0.0212

ERROR = 10%

Y_{11} = 0.61027E-02+j0.61256E-03	Y_{12} = 0.83726E-03+j0.89092E-03
Y_{13} =-0.67588E-03-j0.40582E-03	Y_{14} = 0.90721E-03+j0.13267E-02
Y_{15} = 0.94486E-04+j0.44801E-02	Y_{21} = 0.83727E-03+j0.89092E-03
Y_{22} = 0.61027E-02+j0.61256E-03	Y_{23} = 0.90722E-03+j0.13267E-02
Y_{24} =-0.67587E-03-j0.40582E-03	Y_{25} = 0.94485E-04+j0.44801E-02
Y_{31} =-0.67587E-03-j0.40582E-03	Y_{32} = 0.90722E-03+j0.13267E-02
Y_{33} = 0.62296E-02+j0.14206E-02	Y_{34} = 0.96417E-03+j0.16990E-02
Y_{35} = 0.57071E-04+j0.38180E-02	Y_{41} = 0.90722E-03+j0.13267E-02
Y_{42} =-0.67587E-03-j0.40582E-03	Y_{43} = 0.96417E-03+j0.16990E-02
Y_{44} = 0.62296E-02+j0.14206E-02	Y_{45} = 0.57071E-04+j0.38180E-02
Y_{51} = 0.94486E-04+j0.44801E-02	Y_{52} = 0.94485E-04+j0.44801E-02
Y_{53} = 0.57071E-04+j0.38180E-02	Y_{54} = 0.57071E-04+j0.38180E-02
Y_{55} = 0.45785E-03-j0.15949E-02	

Z_{11} = 0.12052E+03+j0.94596E+01	Z_{12} =-0.53936E+02-j0.32824E+02
Z_{13} =-0.14910E+02+j0.38778E+02	Z_{14} =-0.49643E+02-j0.33173E+02
Z_{15} = 0.14639E+02-j0.58051E+02	Z_{21} =-0.53936E+02-j0.32824E+02
Z_{22} = 0.12052E+03+j0.94597E+01	Z_{23} =-0.49643E+02-j0.33173E+02
Z_{24} =-0.14910E+02+j0.38778E+02	Z_{25} = 0.14639E+02-j0.58051E+02
Z_{31} =-0.14910E+02+j0.38778E+02	Z_{32} =-0.49643E+02-j0.33173E+02
Z_{33} = 0.12794E+03+j0.80123E+01	Z_{34} =-0.46506E+02-j0.34271E+02
Z_{35} =-0.22818E+00-j0.46143E+02	Z_{41} =-0.49643E+02-j0.33173E+02
Z_{42} =-0.14910E+02+j0.38778E+02	Z_{43} =-0.46506E+02-j0.34271E+02
Z_{44} = 0.12794E+03+j0.80123E+01	Z_{45} =-0.22827E+00-j0.46143E+02
Z_{51} = 0.14639E+02-j0.58051E+02	Z_{52} = 0.14639E+02-j0.58051E+02
Z_{53} =-0.22821E+00-j0.46143E+02	Z_{54} =-0.22827E+00-j0.46143E+02
Z_{55} = 0.86315E+02+j0.53452E+02	

Table D9: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0067
R = 0.3000
X_5 = 0.0060
Y_5 = 0.0000

ERROR = 2%

Y_{11} = 0.65745E-02-j0.29000E-02	Y_{12} = 0.97910E-03+j0.13669E-02
Y_{13} =-0.72367E-03-j0.40158E-03	Y_{14} = 0.97910E-03+j0.13669E-02
Y_{15} = 0.14269E-03+j0.45175E-02	Y_{21} = 0.97910E-03+j0.13669E-02
Y_{22} = 0.65960E-02-j0.27761E-02	Y_{23} = 0.10003E-02+j0.14868E-02
Y_{24} =-0.72399E-03-j0.40029E-03	Y_{25} = 0.13623E-03+j0.44180E-02
Y_{31} =-0.72367E-03-j0.40158E-03	Y_{32} = 0.10003E-02+j0.14868E-02
Y_{33} = 0.66170E-02-j0.26600E-02	Y_{34} = 0.10003E-02+j0.14868E-02
Y_{35} = 0.12928E-03+j0.43226E-02	Y_{41} = 0.97910E-03+j0.13669E-02
Y_{42} =-0.72399E-03-j0.40029E-03	Y_{43} = 0.10003E-02+j0.14868E-02
Y_{44} = 0.65960E-02-j0.27761E-02	Y_{45} = 0.13623E-03+j0.44180E-02
Y_{51} = 0.14269E-03+j0.45175E-02	Y_{52} = 0.13623E-03+j0.44180E-02
Y_{53} = 0.12928E-03+j0.43226E-02	Y_{54} = 0.13623E-03+j0.44180E-02
Y_{55} = 0.48506E-03-j0.53367E-02	

Z_{11} = 0.88838E+02+j0.37370E+02	Z_{12} =-0.10606E+02-j0.41101E+02
Z_{13} =-0.34531E+02-j0.25428E+01	Z_{14} =-0.10606E+02-j0.41101E+02
Z_{15} = 0.24719E+02-j0.41721E+02	Z_{21} =-0.10606E+02-j0.41101E+02
Z_{22} = 0.89063E+02+j0.37573E+02	Z_{23} =-0.10379E+02-j0.40911E+02
Z_{24} =-0.34530E+02-j0.25418E+01	Z_{25} = 0.22761E+02-j0.41855E+02
Z_{31} =-0.34531E+02-j0.25428E+01	Z_{32} =-0.10379E+02-j0.40911E+02
Z_{33} = 0.89291E+02+j0.37750E+02	Z_{34} =-0.10379E+02-j0.40911E+02
Z_{35} = 0.20856E+02-j0.41918E+02	Z_{41} =-0.10606E+02-j0.41101E+02
Z_{42} =-0.34530E+02-j0.25418E+01	Z_{43} =-0.10379E+02-j0.40911E+02
Z_{44} = 0.87063E+02+j0.37573E+02	Z_{45} = 0.22761E+02-j0.41855E+02
Z_{51} = 0.24719E+02-j0.41721E+02	Z_{52} = 0.22761E+02-j0.41855E+02
Z_{53} = 0.20856E+02-j0.41918E+02	Z_{54} = 0.22761E+02-j0.41855E+02
Z_{55} = 0.74842E+02+j0.39682E+02	

Table D10: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

$A = 0.0123$
$R = 0.3000$
$X_5 = 0.0060$
$Y_5 = 0.0000$

ERROR = 2%

Y_{11} = 0.63980E-02-j0.19501E-02	Y_{12} = 0.95574E-03+j0.13228E-02
Y_{13} =-0.70168E-03-j0.39097E-03	Y_{14} = 0.95574E-03+j0.13228E-02
Y_{15} = 0.14172E-03+j0.43936E-02	Y_{21} = 0.95574E-03+j0.13228E-02
Y_{22} = 0.64191E-02-j0.18293E-02	Y_{23} = 0.97660E-03+j0.14398E-02
Y_{24} =-0.70199E-03-j0.38970E-03	Y_{25} = 0.13565E-03+j0.42967E-02
Y_{31} =-0.70168E-03-j0.39097E-03	Y_{32} = 0.97661E-03+j0.14398E-02
Y_{33} = 0.64397E-02-j0.17161E-02	Y_{34} = 0.97661E-03+j0.14398E-02
Y_{35} = 0.12908E-03+j0.42038E-02	Y_{41} = 0.95574E-03+j0.13228E-02
Y_{42} =-0.70199E-03-j0.38970E-03	Y_{43} = 0.97660E-03+j0.14398E-02
Y_{44} = 0.64191E-02-j0.18293E-02	Y_{45} = 0.13565E-03+j0.42967E-02
Y_{51} = 0.14172E-03+j0.43936E-02	Y_{52} = 0.13565E-03+j0.42967E-02
Y_{53} = 0.12908E-03+j0.42038E-02	Y_{54} = 0.13565E-03+j0.42967E-02
Y_{55} = 0.47422E-03-j0.43123E-02	

Z_{11} = 0.97609E+02+j0.34704E+02	Z_{12} =-0.17955E+02-j0.42433E+02
Z_{13} =-0.36886E+02+j0.76726E+01	Z_{14} =-0.17955E+02-j0.42433E+02
Z_{15} = 0.21457E+02-j0.45055E+02	Z_{21} =-0.17955E+02-j0.42433E+02
Z_{22} = 0.98030E+02+j0.34948E+02	Z_{23} =-0.17537E+02-j0.42212E+02
Z_{24} =-0.36884E+02+j0.76739E+01	Z_{25} = 0.19286E+02-j0.44846E+02
Z_{31} =-0.36886E+02+j0.76726E+01	Z_{32} =-0.17537E+02-j0.42212E+02
Z_{33} = 0.98445E+02+j0.35147E+02	Z_{34} =-0.17537E+02-j0.42212E+02
Z_{35} = 0.17190E+02-j0.44560E+02	Z_{41} =-0.17955E+02-j0.42433E+02
Z_{42} =-0.36884E+02+j0.76739E+01	Z_{43} =-0.17537E+02-j0.42212E+02
Z_{44} = 0.98030E+02+j0.34948E+02	Z_{45} = 0.19286E+02-j0.44846E+02
Z_{51} = 0.21457E+02-j0.45055E+02	Z_{52} = 0.19286E+02-j0.44846E+02
Z_{53} = 0.17190E+02-j0.44560E+02	Z_{54} = 0.19286E+02-j0.44846E+02
Z_{55} = 0.76077E+02+j0.42386E+02	

Table D11: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

A = 0.0185
R = 0.3000
X_5= 0.0060
Y_5= 0.0000

ERROR = 2%

Y_{11} = 0.35926E-03+j0.10541E-01	Y_{12} =-0.20373E-04-j0.18020E-03
Y_{13} =-0.13368E-03-j0.45604E-04	Y_{14} =-0.20373E-04-j0.18020E-03
Y_{15} = 0.12006E-03-j0.19757E-03	Y_{21} =-0.20373E-04-j0.18020E-03
Y_{22} = 0.35988E-03+j0.10541E-01	Y_{23} =-0.19759E-04-j0.17983E-03
Y_{24} =-0.13367E-03-j0.45608E-04	Y_{25} = 0.11284E-03-j0.19710E-03
Y_{31} =-0.13368E-03-j0.45604E-04	Y_{32} =-0.19759E-04-j0.17953E-03
Y_{33} = 0.36049E-03+j0.10542E-01	Y_{34} =-0.19759E-04-j0.17983E-03
Y_{35} = 0.10577E-03-j0.19656E-03	Y_{41} =-0.20373E-04-j0.18020E-03
Y_{42} =-0.13367E-03-j0.45608E-04	Y_{43} =-0.19759E-04-j0.17983E-03
Y_{44} = 0.35988E-03+j0.10541E-01	Y_{45} = 0.11284E-03-j0.19710E-03
Y_{51} = 0.12006E-03-j0.19757E-03	Y_{52} = 0.11284E-03-j0.19710E-03
Y_{53} = 0.10577E-03-j0.19656E-03	Y_{54} = 0.11284E-03-j0.19710E-03
Y_{55} = 0.32123E-03+j0.10546E-01	

Z_{11} = 0.32539E+01-j0.94822E+02	Z_{12} =-0.70889E-01-j0.16640E+01
Z_{13} =-0.11370E+01-j0.56803E+00	Z_{14} =-0.70889E-01-j0.16640E+01
Z_{15} = 0.12124E+01-j0.17881E+01	Z_{21} =-0.70888E-01-j0.16640E+01
Z_{22} = 0.32569E+01-j0.94819E+02	Z_{23} =-0.67864E-01-j0.16617E+01
Z_{24} =-0.11370E+01-j0.56805E+00	Z_{25} = 0.11478E+01-j0.17887E+01
Z_{31} =-0.11370E+01-j0.56803E+00	Z_{32} =-0.67862E-01-j0.16617E+01
Z_{33} = 0.32599E+01-j0.94817E+02	Z_{34} =-0.67862E-01-j0.16617E+01
Z_{35} = 0.10844E+01-j0.17887E+01	Z_{41} =-0.70888E-01-j0.16640E+01
Z_{42} =-0.11370E+01-j0.56805E+00	Z_{43} =-0.67864E-01-j0.16617E+01
Z_{44} = 0.32569E+01-j0.94819E+02	Z_{45} = 0.11478E+01-j0.17887E+01
Z_{51} = 0.12124E+01-j0.17881E+01	Z_{52} = 0.11477E+01-j0.17887E+01
Z_{53} = 0.10844E+01-j0.17887E+01	Z_{54} = 0.11478E+01-j0.17887E+01
Z_{55} = 0.30503E+01-j0.94813E+02	

Table D-12: Short Circuit Admittance (Y's) and Open Circuit Impedance (Z's) of Circular Array Elements and the Center Element. Values Shown Follow the Order: Real, and Imaginary.

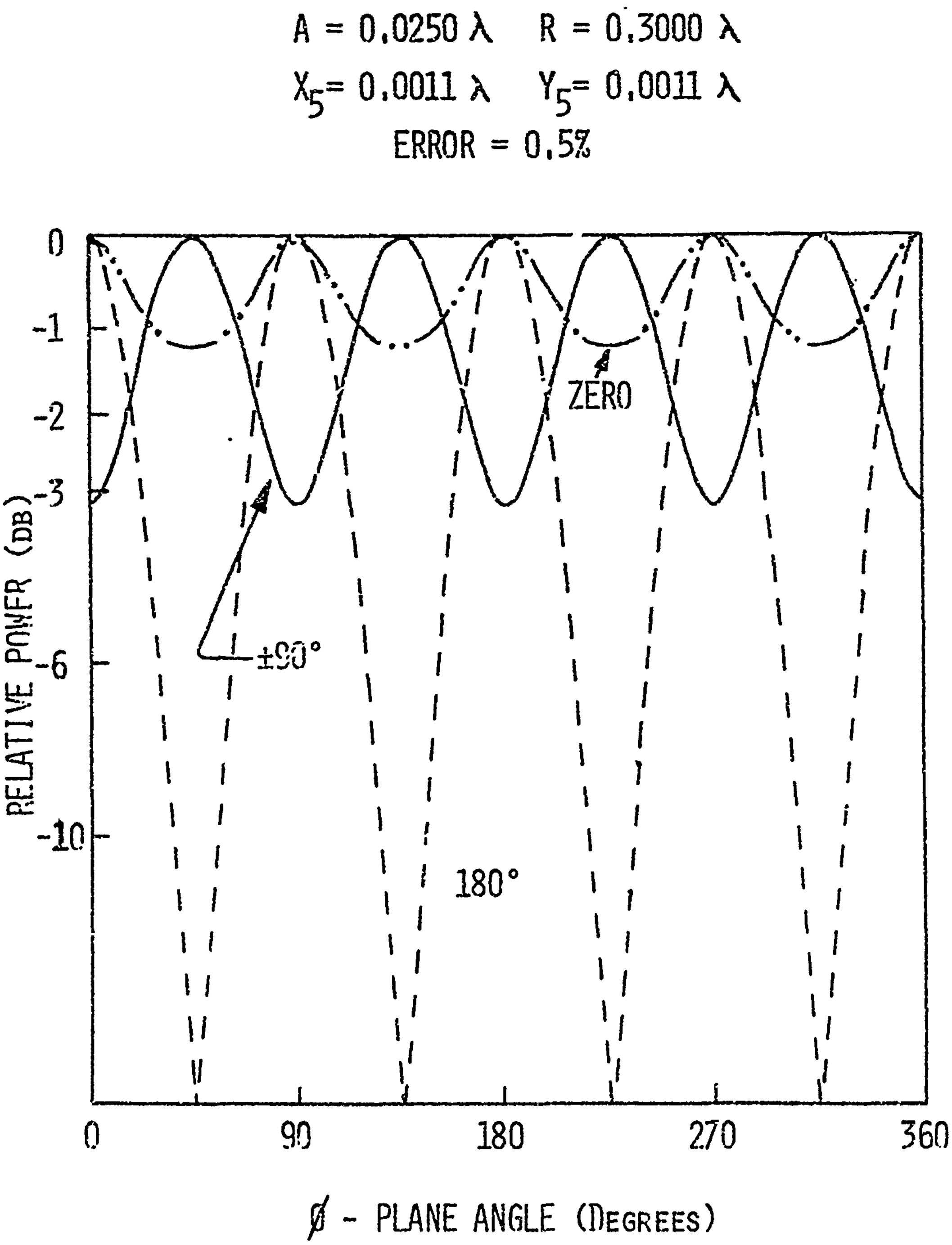


Figure D-7 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

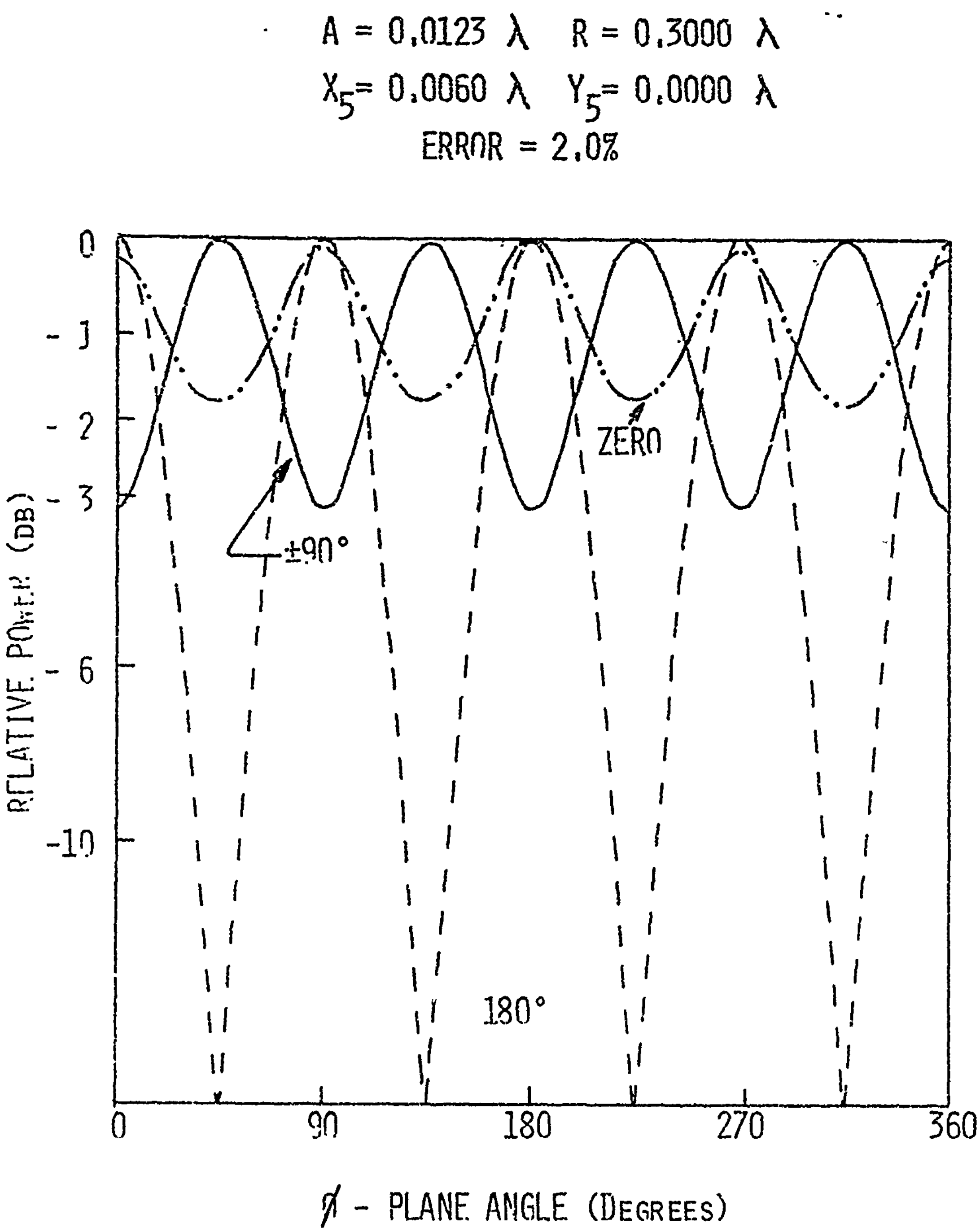


Figure D-8 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

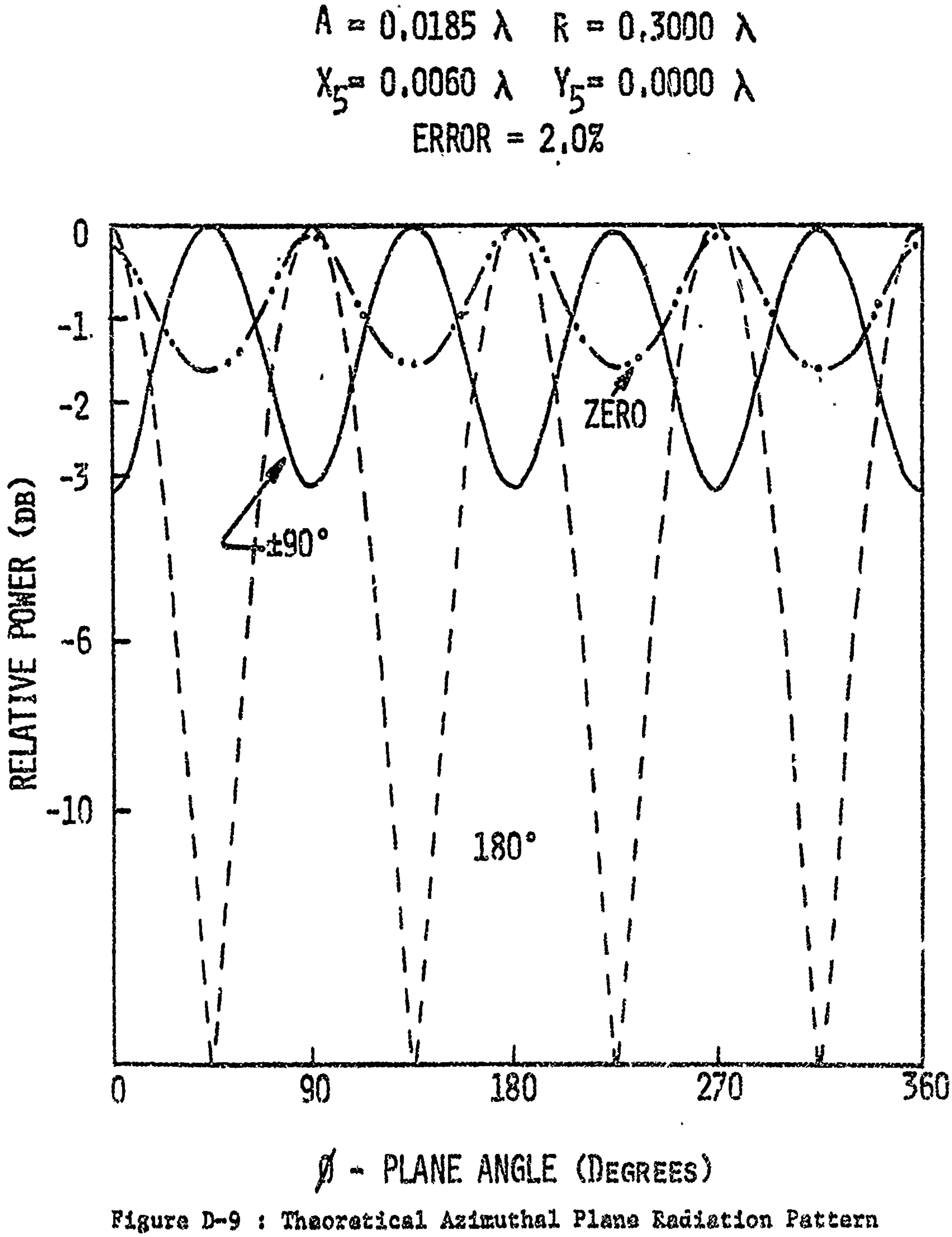


Figure D-9 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.

$A = 0.0067\ \lambda \quad R = 0.3000\ \lambda$

$X_5 = 0.0060\ \lambda \quad Y_5 = 0.0000\ \lambda$

ERROR = 2.0%

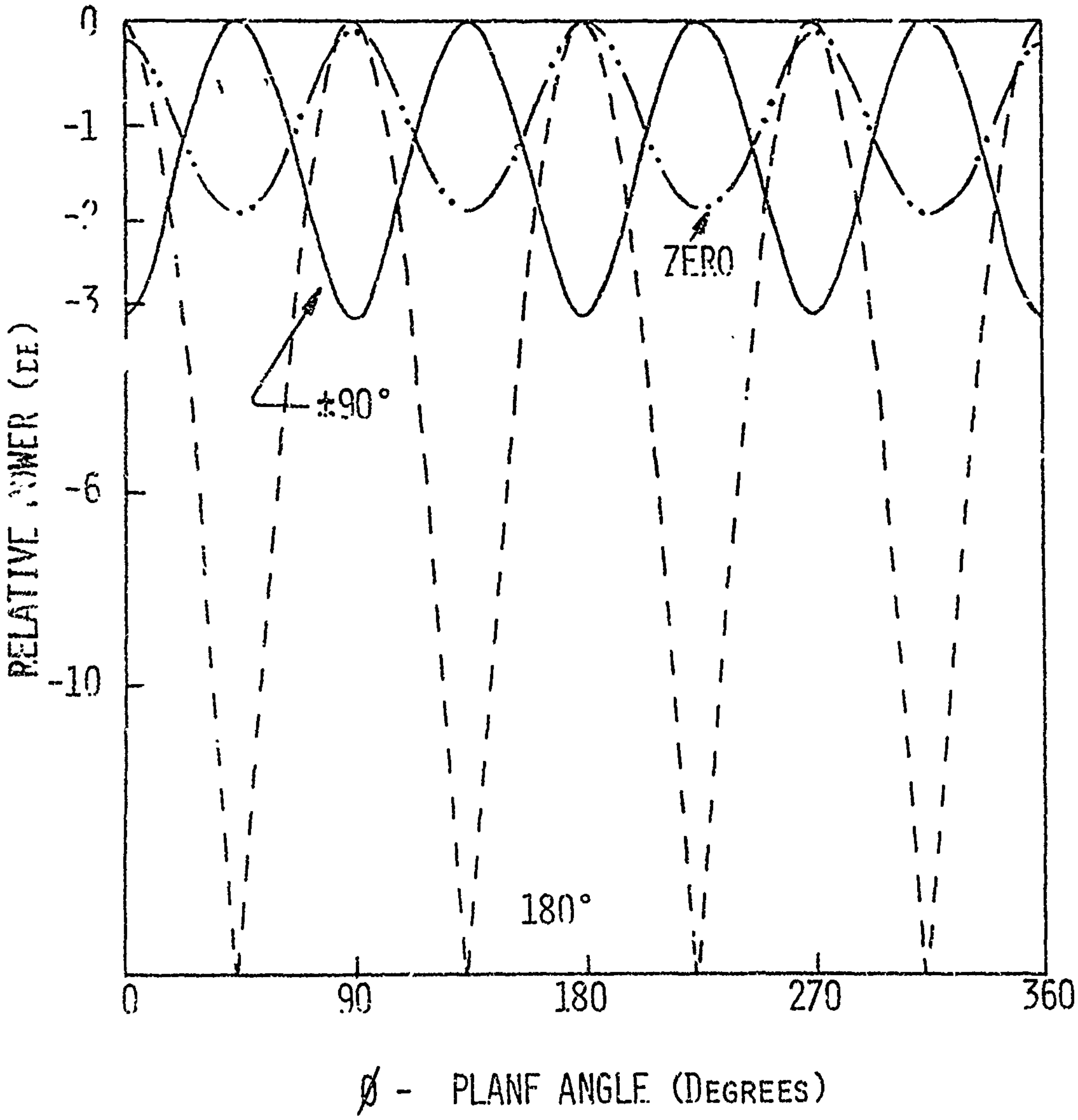


Figure D-10 : Theoretical Azimuthal Plane Radiation Pattern of the Circular Array.